21-st International Olympiad In Informatics

TASKS AND SOLUTIONS

August 8 - 15, 2009

Plovdiv, Bulgaria

Day 0

Task 0.1 AREA

A large agricultural corporation is undertaking an audit of its N land properties. As a first step, they would like to know the total area of their properties. Each property has a rectangular shape, and no two properties overlap. You realize that this is a trivial calculation, but you understand that you need to solve this problem in order to test your ability to properly handle large amounts of data and large numbers under the IOI 2009 contest system.

TASK

Write a program that, given the dimensions of the N land properties, determines the sum of their areas.

CONSTRAINTS

$1 \leq N \leq 500\ 000$ – The number of land properties
$1 \leq A_k, B_k \leq 20\ 000$ – The lengths of the sides of property k

INPUT

Your program should read from the standard input the following data:

- The first line contains a single integer: the number of properties N.
- The next N lines describe the properties, one property per line. The k^{th} of these lines describes property number k and it contains two integers separated by a single space: the lengths of the sides of the property A_k and B_k measured in meters.

OUTPUT

Your program should write to the standard output a single line containing a single integer: the total area of all properties, measured in square meters.

IMPORTANT NOTE

The final answer may not fit in 32 bits. You have to use a 64-bit data type, such as `long long` in C/C++ or `int64` in Pascal, in order to compute and store the answer in a single variable. Please see the technical info sheet for details.

EXAMPLE

Sample Input	Sample Output
3 15 25 5 6 3 1	408

Task 0.2 HILL (proposed by Iskren Chernev)

An extraterrestrial team of contestants, known as the aliens, is coming to Plovdiv for the IOI. They would like to land their flying saucer on one of Plovdivs hills, but they have no map of the city, so they need to use their laser scanner to find a hill. The aliens have divided the city into a grid of N by M cells, and each time they use the scanner they can determine the altitude of a single cell. Their scanner is so precise that no two cells would ever have the same altitude.

The aliens define a hill as a cell that has higher altitude than all of its adjacent cells. Two cells are considered adjacent if they share a side. Thus typically each cell has four adjacent ones, with the cells on the border of the grid being the exception (they have less than four adjacent cells).

The aliens have time for only 3 050 scans before their saucer hits the ground. Help the aliens find a hill before they run out of time.

TASK

Write a program that, given the dimensions of the grid, interactively uses the scanner to determine the location of a hill, while doing no more than 3 050 scans in total.

CONSTRAINTS

$1 \leq N, M \leq 1\ 000$ – The dimensions of the grid

$1 \leq A_{x,y} \leq 2\ 000\ 000\ 000$ – The altitude of cell $< x, y >$

INPUT

Your program should read from the standard input a single line with two integers N and M, separated by a single space.

INTERACTION

After reading the input data, your program should start using the laser scanner. Each time you want to use the scanner for a cell with coordinates x and y ($1 \leq x \leq N$, $1 \leq y \leq M$), you should print a single line on the standard output, containing three integers separated by single spaces: 0 (a literal zero), x and y, in this order.

After submitting your request to the scanner, you should read a single line on the standard input. It will contain a single integer: the altitude of the cell with coordinates x and y in some alien units.

OUTPUT

When your program has determined the location of a hill $< a, b >$ ($1 \leq a \leq N$, $1 \leq b \leq M$), you should report your answer by printing a single line on the standard output, containing three integers separated by single spaces: 1 (a literal one), a and b, in this order.

IMPORTANT NOTE: In order to interact properly with the scanner, your program needs to flush the standard output after every line that you print on the standard output. Please see the technical info sheet for instructions on how to do this properly.

EXAMPLE

Sample Input	Sample Output
3 2	
	0 1 1 [flush stdout/output]
3	
	0 2 1 [flush stdout/output]
4	
	0 3 1 [flush stdout/output]
1	
	0 1 2 [flush stdout/output]
7	
	0 2 2 [flush stdout/output]
6	
	1 1 2 [flush stdout/output]

0.3 MUSEUM (proposed by Boyko Bantchev)

The Plovdiv Museum of Modern Art has an exhibition of ancient Thracian vases. There are N vases total. The first one is a miniature of height 1 centimeter. The second one is of height 2 centimeters; the third one is 3 centimeters tall and so on until the N^{th} vase, which is N centimeters tall.

Since this a modern art museum and the vases are ancient, the organizers of the exhibition would like to add a modern, chaotic twist to the presentation of the vases. They have decided to arrange the vases in a line that satisfies the following condition: For any three vases A, B and C, such that Bs height is exactly the average of the heights of A and C, either B must be positioned to the left of both A and C, or B must be positioned to the right of both A and C (in other words, B may not be positioned between A and C on the line).

TASK

Write a program that, given the number of vases, determines a linear arrangement of the vases that satisfies the condition of the exhibition organizers.

CONSTRAINTS

$1 \leq N \leq 2\ 000$ – The number of vases

INPUT

You are given five problem instances in the files `museum.1.in` to `museum.5.in`. Each file contains a single line, which in turn contains a single integer: the number of vases N.

OUTPUT

You are to submit five output files, named `museum.1.out` to `museum.5.out`, each corresponding to one of the input files. The files should be in the following format:

There should be N lines, each representing the N positions in the arrangement, in order from left to right. Line k should contain a single integer H_k, the height of the vase you decided to place on position k. All N heights should be distinct integers between 1 and N inclusive.

EXAMPLE

Sample Input `museum.0.in`	Sample Output `museum.0.out`
5	3 1 2 5 4

In the above arrangement, 3 is neither between 2 and 4, nor is it between 1 and 5. Also, 2 is not between 1 and 3, and 4 is not between 3 and 5. Thus, it satisfies the condition of the exhibition organizers.

TECHNICAL INFO SHEET (day 0)

These pages contain helpful information on how to avoid slow input/output performance with C++ streams (cin / cout), how to use 64-bit data types (variables) and how to flush the output for interactive tasks.

Slow Input / Output with C++ Streams

When solving tasks with very large amounts of input / output data, you may notice that C++ programs using the cin and cout streams are much slower than equivalent programs that use the scanf and printf functions for input and output processing. Thus, if you are using the cin / cout streams we strongly recommend that you switch to using scanf / printf instead. However, if you still want to use cin / cout, we recommend adding the following line at the beginning of your program:

```
ios::sync_with_stdio(false);
```

and also making sure that you never use `endl`, but use `\n` instead.

Please note, however, that including `ios::sync_with_stdio(false)` breaks the synchrony between cin / cout and scanf / printf, so if you are using this, you should never mix usage of cin and scanf, nor mix cout and printf.

64-bit Data Types

For some tasks you may need to deal with numbers too large to fit in 32 bits. In these cases, you would have to use a 64-bit integer data type, such as long long in C/C++ or int64 in Pascal. Here is some sample code that illustrates the usage of these data types:

C/C++

```
int main(void) {
    long long varname;
    scanf("%lld", &varname);
    // Do something with the varname variable
    printf("%lld\n", varname);
    return 0;
}
```

Pascal

```
var
    varname: Int64;
begin
    read(varname);
    { Do something with the varname variable }
    writeln(varname);
end.
```

Flushing the Output

Whenever you solve an interactive task, you always need to flush the buffer of your output after every new line printed on the output. Here is some code to illustrate how to do this under C, C++ and Pascal:

C or C++ with scanf / printf

```
fflush(stdout);
```

C++ with cin / cout

```
cout << flush;
```

Pascal

```
flush(output);
```

Task Overview Sheet (Day 0)

	Area	Hill	Museum
Type	Batch	Interactive	Output-only
Detailed Feedback	Full	Partial	None
Time Limit (per test case)	2 seconds	1 second	Not Applicable
Memory Limit (per test case)	32 MB	64 MB	Not Applicable
Points	100	100	100

NOTE: On the actual competition there will be four problems on each day. We have only three problems here because the practice contest is shorter in duration and because there are only three possible task types.

Day 1

1.1 ARCHERY (proposed by Velin Tzanov)

An archery tournament is held according to the following rules. There are N targets arranged in a line and numbered from 1 to N inclusive according to their place on the line (the leftmost target being target 1, and the rightmost target being target N). There are also $2*N$ archers. At any time during the tournament, there are two archers on each target. Every round of the tournament goes according to the following procedure:

The two archers on each target compete with each other and determine a winner and a loser between them. Then all archers are rearranged as follows:

The winners on targets 2 to N inclusive move to the target on their left (i.e., targets 1 to $N-1$ respectively).

The losers on targets 2 to N inclusive, as well as the winner on target 1, remain on the same target.

The loser on target 1 moves to target N.

The tournament continues for R rounds, with the number of rounds being at least as many as the number of archers (i.e., $R \geq 2*N$).

You are the only archer to arrive for the tournament exactly on time. All other $2*N-1$ archers have arrived early and are already standing in a line. What you have to do now is to insert yourself somewhere into the line amongst them. You know that after you take your position, the two leftmost archers in the line will start the tournament on target 1, the next two will start on target 2 and so on, with the two rightmost archers starting on target N.

All the $2*N$ archers in the tournament (including yourself) are ranked by skill, where a smaller rank corresponds to better skill. No two archers have the same rank. Also, whenever two archers compete, the one with the smaller rank will always win.

Knowing how skilled each of your competitors is, you want to insert yourself in such a way as to ensure that you will finish the tournament on a target with as small a number as possible. If there are multiple ways to do this, you prefer the one that starts at a target with as large a number as possible.

TASK

Write a program that, given the ranks of all archers, including yourself, as well as your competitors arrangement on the line, determines on which target you should start the tournament, so that you can achieve your goals as defined above.

CONSTRAINTS

$1 \leq N \leq 200\ 000$ – The number of targets; also equal to half the number of archers
$2 * N \leq R \leq 1\ 000\ 000\ 000$ – The number of tournament rounds
$1 \leq S_k \leq 2 * N$ – The rank of archer k

INPUT

Your program must read from standard input the following data:

The first line contains the integers N and R, separated by a space.

The next $2 * N$ lines list the ranks of the archers. The first of these lines contains your rank. The rest of these lines contain the ranks of the other archers, one archer per line, in the order in which they have arranged themselves (from left to right). Each of these $2 * N$ lines contains a single integer between 1 and $2 * N$ inclusive. A rank of 1 is the best and a rank of $2 * N$ is the worst. No two archers have the same rank.

OUTPUT

Your program must write to standard output a single line containing a single integer between 1 and N inclusive: the number of the target on which you will start the tournament.

GRADING

For a number of tests, worth a total of 60 points, N will not exceed 5 000. Also, for some of these tests, worth a total of 20 points, N will not exceed 200.

EXAMPLES

Sample Input	Smaple Output
4 8 7 4 2 6 5 8 1 3	3

You are the second worst archer. If you start on target 1, you will then go to target 4 and stay there until the end. If you start on target 2 or 4, you will just stay there for the whole tournament. If you start on target 3, you will beat the worst archer and then move to target 2 and stay there.

Sample Input	Smaple Output
4 9 2 1 5 8 3 4 7 6	2

You are the second best archer. The best one is already on target 1 and will stay there for the whole duration of the tournament. Thus, no matter where you start, you will always move from your target, going through all targets from 4 to 1 over and over again. In order for you to end on target 1 after 9 transitions, you have to start on target 2.

1.2 HIRING (proposed by Velin Tzanov)

You have to hire workers for a construction project. There are N candidates applying for the job, numbered from 1 to N inclusive. Each candidate k requires that if he is hired, he must be paid at least S_k dollars. Also, each candidate k has a qualification level Q_k. The regulations of the construction industry require that you pay your workers in proportion to their qualification level, relative to each other. For example, if you hire two workers A and B, and $Q_A = 3 * Q_B$, then you have to pay worker A exactly three times as much as you pay worker B. You are allowed to pay your workers non-integer amounts of money. This even includes quantities that cannot be written with a finite number of digits in decimal form, such as a third or a sixth of a dollar.

You have W dollars at hand and you want to hire as many workers as possible. You decide whom to hire and how much to pay them, but you have to meet the minimum salary requirements of those you choose to hire, and you have to obey the industry regulations. You also have to fit within your budget of W dollars.

The nature of your project is such that the qualification level is completely irrelevant, so you are only interested in maximizing the number of workers without regard to their qualification level. However, if there is more than one way to achieve this, then you want to select the one where the total amount of money you have to pay your workers is as small as possible. In case there is more than one way to achieve this, then you are indifferent among these ways and you would be satisfied with any one of them.

TASK

Write a program that, given the different salary requirements and qualification levels of the candidates, as well as the amount of money you have, determines which candidates you should hire. You must hire as many of them as possible and you must do so with as little money as possible, while complying with the industry regulations specified above.

CONSTRAINTS

$1 \leq N \leq 500\ 000$ – The number of candidates
$1 \leq S_k \leq 20\ 000$ – The minimum salary requirement of candidate k
$1 \leq Q_k \leq 20\ 000$ – The qualification level of candidate k
$1 \leq W \leq 10\ 000\ 000\ 000$ – The amount of money available to you

IMPORTANT NOTE

The maximum value of W does not fit in 32 bits. You have to use a 64-bit data type, such as long long in C/C++ or `int64` in Pascal, in order to store the value of W in a single variable. Please see the technical info sheet for details.

INPUT

Your program must read from standard input the following data:

• The first line contains the integers N and W, separated by a space.
• The next N lines describe the candidates, one candidate per line. The k^{th} of these lines describes candidate number k and it contains the integers S_k and Q_k, separated by a space.

OUTPUT

Your program must write to standard output the following data:
• The first line must contain a single integer H, the number of workers that you hire.
• The next H lines must list the identifying numbers of the candidates you choose to hire (each of them a different number between 1 and N), one per line, in any order.

GRADING

For any given test case, you will receive full points if your choice of candidates enables you to achieve all of your goals, while satisfying all constraints. If you produce an output file with a correct first line (i.e., a correct value of H), but which does not meet the above description, you will receive 50% of the points for that test case. The latter will be the case even if the output file is not properly formatted, as long as the first line is correct.

For a number of tests, worth a total of 50 points, N will not exceed 5 000.

EXAMPLES

Sample Input	Sample Output
4 100 5 1000 10 100 8 10 20 1	2 2 3

The only combination for which you can afford to hire two workers and still meet all the constraints is if you select workers 2 and 3. You can pay them 80 and 8 dollars respectively and thus fit in your budget of 100.

Sample Input	Sample Output
3 4 1 2 1 3 1 3	3 1 2 3

Here you can afford to hire all three workers. You pay 1 dollar to worker 1 and 1.50 dollars each to workers 2 and 3, and you manage to hire everyone with the 4 dollars that you have.

Sample Input	Sample Output
3 40 10 1 10 2 10 3	2 2 3

Here you cannot afford to hire all three workers, as it would cost you 60 dollars, but you can afford to hire any two of them. You choose to hire workers 2 and 3 because they would cost you the smallest sum of money, compared to the other two-worker combinations. You can pay 10 dollars to worker 2 and 15 dollars to worker 3 for a total of 25 dollars. If you were to hire workers 1 and 2 you would have to pay them at least 10 and 20 dollars respectively. If you were to hire 1 and 3, then you would have to pay them at least 10 and 30 dollars respectively.

Task 1.3 POI (proposed by Carl Hultquist)

The local Plovdiv Olympiad in Informatics (POI) was held according to the following unusual rules. There were N contestants and T tasks. Each task was graded with only one test case, therefore for every task and every contestant there were only two possibilities: either the contestant solved the task, or the contestant did not solve the task. There was no partial scoring on any task.

The number of points assigned to each task was determined after the contest and was equal to the number of contestants that did not solve the task. The score of each contestant was equal to the sum of points assigned to the tasks solved by that contestant.

Philip participated in the contest, but he is confused by the complicated scoring rules, and now he is staring at the results, unable to determine his place in the final standings. Help Philip by writing a program that calculates his score and his ranking.

Before the contest, the contestants were assigned unique IDs from 1 to N inclusive. Philips ID was P. The final standings of the competition list the contestants in descending order of their scores. In case of a tie, among the tied contestants, those who have solved more tasks will be listed ahead of those who have solved fewer tasks. In case of a tie by this criterion as well, the contestants with equal results will be listed in ascending order of their IDs.

TASK

Write a program that, given which problems were solved by which contestant, determines Philips score and his rank in the final standings.

CONSTRAINTS

$1 \leq N \leq 2\ 000$ – The number of contestants
$1 \leq T \leq 2\ 000$ – The number of tasks
$1 \leq P \leq N$ – Philips ID

INPUT

Your program must read from standard input the following data:

• The first line contains the integers N, T and P, separated by individual spaces.

• The next N lines describe which tasks were solved by which contestant. The k^{th} of these lines describes which tasks were solved by the contestant with ID k. Each such line contains T integers, separated by spaces. The first of these numbers denotes whether or not contestant k solved the first task. The second number denotes the same for the second task and so on. These T numbers are all either 0 or 1, where 1 means that contestant k solved the corresponding task, and 0 means that he or she did not solve it.

OUTPUT

Your program must write to standard output a single line with two integers separated by a single space. First, the score that Philip got on the POI competition. Second, Philips rank in the final standings. The rank is an integer between 1 and N inclusive, with 1 denoting the contestant listed at the top (i.e., a contestant who has the highest score) and N to the one listed at the bottom (i.e., a contestant with the lowest score).

GRADING

For a number of tests, worth a total of 35 points, no other contestant will have the same score as Philip.

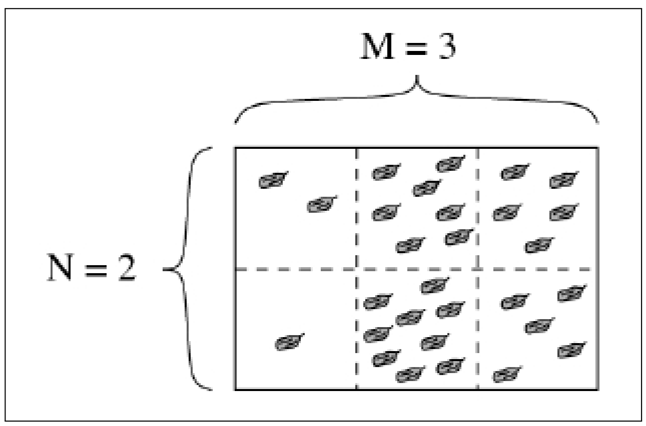
EXAMPLE

Sample Input	Sample Output
5 3 2 0 0 1 1 1 0 1 0 0 1 1 0 1 1 0	3 2

The first problem was unsolved by only one contestant, so it is worth 1 point. The second problem was unsolved by two contestants, so it is worth 2 points. The third problem was unsolved by four contestants, so it is worth 4 points. Thus the first contestant has a score of 4; the second contestant (Philip), the fourth and the fifth contestants all have a score of 3; and the third contestant has a score of 1. Contestants 2, 4 and 5 are all tied according to the first tie-break rule (number of problems solved), and according to the second tie-break rule (smaller ID) Philip ranks before the others. Thus Philips rank in the final standings is 2. He is only behind the contestant with ID 1.

Task 1.4 RAISINS (proposed by Emil Kelevedjiev)

Plovdiv's famous master chocolatier Bonny needs to cut a slab of chocolate with raisins. The chocolate is a rectangular block of identical square pieces. The pieces are aligned with the edges of the chocolate, and they are arranged in N rows and M columns, for a total of $N * M$ pieces. Each piece has one or more raisins on it, and no raisins lie between or across pieces.



Initially, the chocolate is one single, monolithic block. Bonny needs to cut it into smaller and smaller blocks until finally she has cut the chocolate down to its $N*M$ individual pieces. As Bonny is very busy, she needs the help of her assistant, Sly Peter, to do the cutting. Peter only makes straight line, end-to-end cuts and he wants to be paid for every single cut he makes. Bonny has no money at hand, but she has plenty of raisins left over, so she offers to pay Peter in raisins. Sly Peter agrees to this arrangement, but under the following condition: every time he cuts a given block of chocolate into two smaller blocks, he has to be paid as many raisins as there are on the block he was given.

Bonny wants to pay Peter as little as possible. She knows how many raisins there are on each of the $N * M$ pieces. She can choose the order in which she gives Peter any remaining blocks, and she can also tell Peter what cuts to make (horizontal or vertical) and where exactly to make them. Help Bonny decide how to cut the chocolate into individual pieces, so that she pays Sly Peter as few raisins as possible.

TASK

Write a program that, given the number of raisins on each of the individual pieces, determines the minimum number of raisins that Bonny will have to pay Sly Peter.

CONSTRAINTS

$1 \leq N, M \leq 50$ – The number of pieces on each side of the chocolate
$1 \leq R_{k,p} \leq 1000$ – The number of raisins on the piece in the k^{th} row and the p^{th} column

INPUT

Your program must read from standard input the following data:

• The first line contains the integers N and M, separated by a single space.

• The next N lines describe how many raisins there are on each piece of the chocolate. The k^{th} of these N lines describes the k^{th} row of the chocolate. Each such line contains M integers separated by single spaces. The integers describe the pieces on the corresponding row in order from left to right. The p^{th} integer on the k^{th} line (among these N lines) tells you how many raisins are on the piece in the k^{th} row and the p^{th} column.

OUTPUT

Your program must write to standard output a single line containing a single integer: the minimum possible number of raisins that Bonny would have to pay Sly Peter

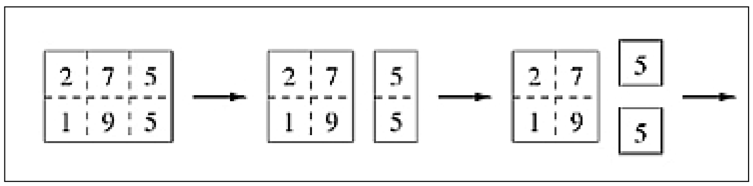
GRADING

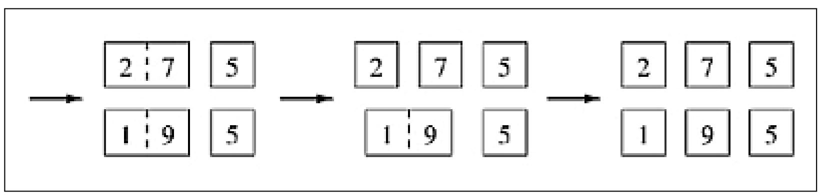
For a number of tests, worth a total of 25 points, N and M will not exceed 7.

EXAMPLE

Sample Input	Sample Output
2 3 2 7 5 1 9 5	77

One possible way (out of many) to achieve a cost of 77 is as follows:





The first cut that Bonny asks Peter to make separates the third column from the rest of the chocolate. Bonny needs to pay Peter 29 raisins for this.

Then Bonny gives Peter the smaller of the two blocks: the one that has two pieces with 5 raisins each, and asks Peter to cut the block in two in exchange for 10 raisins.

After this, Bonny gives Peter the largest remaining block: the one having pieces with 2, 7, 1 and 9 raisins respectively. Bonny asks Peter to cut it horizontally, separating the first and the second row and pays him 19 raisins.

Following this, Bonny gives Peter the top-left block, paying 9 raisins. Finally, Bonny asks Peter to split the bottom-left block, paying 10 raisins.

The total cost to Bonny is 29 + 10 + 19 + 9 + 10 = 77 raisins. No other cutting arrangement can get the chocolate cut into its 6 pieces at a smaller cost.

TECHNICAL INFO SHEET (day 1)

These pages contain helpful information on how to avoid slow input/output performance with C++ streams (cin / cout), how to use 64-bit data types (variables) and how to flush the output for interactive tasks. They also include reference for what options are given to the compilers and what stack limitations are in place.

Slow Input / Output with C++ Streams

When solving tasks with very large amounts of input / output data, you may notice that C++ programs using the cin and cout streams are much slower than equivalent programs that use the scanf and printf functions for input and output processing. Thus, if you are using the cin / cout streams we strongly recommend that you switch to using scanf / printf instead. However, if you still want to use cin / cout, we recommend adding the following line at the beginning of your program:

```
ios::sync_with_stdio(false);
```

and also making sure that you never use `endl`, but use `\n` instead.

Please note, however, that including `ios::sync_with_stdio(false)` breaks the synchrony between cin / cout and scanf / printf, so if you are using this, you should never mix usage of cin and scanf, nor mix cout and printf.

64-bit Data Types

For some tasks you may need to deal with numbers too large to fit in 32 bits. In these cases, you would have to use a 64-bit integer data type, such as `long long` in C/C++ or `int64` in Pascal. Here is some sample code that illustrates the usage of these data types:

C/C++

```
int main(void) {
    long long varname;
    scanf("%lld", &varname);
    // Do something with the varname variable
    printf("%lld\n", varname);
    return 0;
}
```

Pascal

```
var
    varname: Int64;
begin
    read(varname);
    { Do something with the varname variable }
    writeln(varname);
end.
```

Flushing the Output

Whenever you solve an interactive task, you always need to flush the buffer of your output after every new line printed on the output. Here is some code to illustrate how to do this under C, C++ and Pascal:

C or C++ with scanf / printf

```
fflush(stdout);
```

C++ with cin / cout

```
cout << flush;
```

Pascal

```
flush(output);
```

Compiler Options

The following commands will be used to compile solutions of batch and interactive tasks (say the task name is abc):

C

```
gcc -o abc abc.c -std=gnu99 -O2 -s -static -lm -x c
```

C++

```
g++ -o abc abc.cpp -O2 -s -static -lm -x c++
```

Pascal

```
fpc -O2 -XS -Sg abc.pas
```

Stack Limitations

Whenever your program is executed through the contest system, the stack size will only be limited by the memory limit for the corresponding task.

Task Overview Sheet (Day 1)

	Archery	Hiring	POI	Raisins
Type	Batch	Batch	Batch	Batch
Detailed Feedback	Partial	None	Full	Partial
Time Limit (per test case)	2 seconds	1.5 seconds	2 seconds	5 seconds
Memory Limit (per test case)	64 MB	64 MB	64 MB	128 MB
Points	100	100	100	100

NOTE: C++ programmers should be aware that using C++ streams (cin / cout) may lead to I/O bottlenecks and substantially slower performance. Please see the technical info sheet for ways to avoid this.

Day 2

Task 2.1 GARAGE (proposed by Carl Hultquist)

A parking garage has N parking spaces, numbered from 1 to N inclusive. The garage opens empty each morning and operates in the following way throughout the day. Whenever a car arrives at the garage, the attendants check whether there are any parking spaces available. If there are none, then the car waits at the entrance until a parking space is released. If a parking space is available, or as soon as one becomes available, the car is parked in the available parking space. If there is more than one available parking space, the car will be parked at the space with the smallest number. If more cars arrive while some car is waiting, they all line up in a queue at the entrance, in the order in which they arrived. Then, when a parking space becomes available, the first car in the queue (i.e., the one that arrived the earliest) is parked there.

The cost of parking in dollars is the weight of the car in kilograms multiplied by the specific rate of its parking space. The cost does not depend on how long a car stays in the garage.

The garage operator knows that today there will be M cars coming and he knows the order of their arrivals and departures. Help him calculate how many dollars his revenue is going to be today.

TASK

Write a program that, given the specific rates of the parking spaces, the weights of the cars and the order in which the cars arrive and depart, determines the total revenue of the garage in dollars.

CONSTRAINTS

$1 \leq N \leq 100$ – The number of parking spaces
$1 \leq M \leq 2\ 000$ – The number of cars
$1 \leq R_s \leq 100$ – The rate of parking space s in dollars per kilogram
$1 \leq W_k \leq 10\ 000$ – The weight of car k in kilograms

INPUT

Your program must read from standard input the following data:
• The first line contains the integers N and M, separated by a space.

• The next N lines describe the rates of the parking spaces. The s^{th} of these lines contains a single integer R_s, the rate of parking space number s in dollars per kilogram.

• The next M lines describe the weights of the cars. The cars are numbered from 1 to M inclusive in no particular order. The k^{th} of these M lines contains a single integer W_k, the weight of car k in kilograms.

• The next $2 * M$ lines describe the arrivals and departures of all cars in chronological order. A positive integer i indicates that car number i arrives at the garage. A negative integer $-i$ indicates that car number i departs from the garage. No car will depart from the garage before it has arrived, and all cars from 1 to M inclusive will appear exactly twice in this sequence, once arriving and once departing. Moreover, no car will depart from the garage before it has parked (i.e., no car will leave while waiting on the queue).

OUTPUT

Your program must write to standard output a single line containing a single integer: the total number of dollars that will be earned by the garage operator today.

GRADING

For a number of tests worth 40 points there will always be at least one available parking space for every arriving car. In these cases no car will ever have to wait for a space.

EXAMPLES

Sample Input	Sample Output
3 4 2 3 5 200 100 300 800 3 2 −3 1 4 −4 −2 −1	5300

Car number 3 goes to space number 1 and pays $300 * 2 = 600$ dollars.

Car number 2 goes to space number 2 and pays $100 * 3 = 300$ dollars.

Car number 1 goes to space number 1 (which was released by car number 3) and pays $200 * 2 = 400$ dollars.

Car number 4 goes to space number 3 (the last remaining) and pays $800 * 5 =$ 4 000 dollars.

Sample Input	Sample Output
2 4 5 2 100 500 1000 2000 3 1 2 4 −1 −3 −2 −4	16200

Car number 3 goes to space number 1 and pays $1\ 000 * 5 = 5\ 000$ dollars.

Car number 1 goes to space number 2 and pays $100 * 2 = 200$ dollars. Car number 2 arrives and has to wait at the entrance.

Car number 4 arrives and has to wait at the entrance behind car number 2.

When car number 1 releases its parking space, car number 2 parks there and pays $500 * 2 = 1\ 000$ dollars.

When car number 3 releases its parking space, car number 4 parks there and pays $2\ 000 * 5 = 10\ 000$ dollars.

Task 2.2 MECHO (proposed by Carl Hultquist)

Mecho the bear has found a little treasure the bees secret honeypot, which is full of honey! He was happily eating his newfound treasure until suddenly one bee saw him and sounded the bee alarm. He knows that at this very moment hordes of bees will emerge from their hives and start spreading around trying to catch him. He knows he has to leave the honeypot and go home quickly, but the honey is so sweet that Mecho doesnt want to leave too soon. Help Mecho determine the latest possible moment when he can leave.

Mechos forest is represented by a square grid of N by N unit cells, whose sides are parallel to the north-south and east-west directions. Each cell is occupied by a tree, by a patch of grass, by a hive or by Mechos home. Two cells are considered adjacent if one of them is immediately to the north, south, east or west of the other (but not on a diagonal). Mecho is a clumsy bear, so every time he makes a step, it has to be to an adjacent cell. Mecho can only walk on grass and cannot go through trees or hives, and he can make at most S steps per minute.

At the moment when the bee alarm is sounded, Mecho is in the grassy cell containing the honeypot, and the bees are in every cell containing a hive (there may be more than one hive in the forest). During each minute from this time onwards, the following events happen in the following order:

• If Mecho is still eating honey, he decides whether to keep eating or to leave. If he continues eating, he does not move for the whole minute.

• Otherwise, he leaves immediately and takes up to S steps through the forest as described above. Mecho cannot take any of the honey with him, so once he has moved he cannot eat honey again.

After Mecho is done eating or moving for the whole minute, the bees spread one unit further across the grid, moving only into the grassy cells. Specifically, the swarm of bees spreads into every grassy cell that is adjacent to any cell already containing bees. Furthermore, once a cell contains bees it will always contain bees (that is, the swarm does not move, but it grows).

In other words, the bees spread as follows: When the bee alarm is sounded, the bees only occupy the cells where the hives are located. At the end of the first minute, they occupy all grassy cells adjacent to hives (and still the hives themselves). At the end of the second minute, they additionally occupy all grassy cells adjacent to grassy cells adjacent to hives, and so on. Given enough time, the bees will end up simultaneously occupying all grassy cells in the forest that are within their reach.

Neither Mecho nor the bees can go outside the forest. Also, note that according to the rules above, Mecho will always eat honey for an integer number of minutes.

The bees catch Mecho if at any point in time Mecho finds himself in a cell occupied by bees.

TASK

Write a program that, given a map of the forest, determines the largest number of minutes that Mecho can continue eating honey at his initial location, while still being able to get to his home before any of the bees catch him.

CONSTRAINTS

$1 \leq N \leq 800$ – The size (side length) of the map

$1 \leq S \leq 1\ 000$ – The maximum number of steps Mecho can take in each minute

INPUT

Your program must read from standard input the following data:

• The first line contains the integers N and S, separated by a space.

• The next N lines represent the map of the forest. Each of these lines contains N characters with each character representing one unit cell of the grid. The possible characters and their associated meanings are as follows:

T denotes a tree

G denotes a grassy cell

M denotes the initial location of Mecho and the honeypot, which is also a grassy cell

D denotes the location of Mechos home, which Mecho can enter, but the bees cannot.

H denotes the location of a hive

NOTE: It is guaranteed that the map will contain exactly one letter M, exactly one letter D and at least one letter H. It is also guaranteed that there is a sequence of adjacent letters G that connects Mecho to his home, as well as a sequence of adjacent letters G that connects at least one hive to the honeypot (i.e., to Mechos initial location). These sequences might be as short as length zero, in case Mechos home or a hive is adjacent to Mechos initial location. Also, note that the bees cannot pass through or fly over Mechos home. To them, it is just like a tree.

OUTPUT

Your program must write to standard output a single line containing a single integer: the maximum possible number of minutes that Mecho can continue eating honey at his initial location, while still being able to get home safely.

If Mecho cannot possibly reach his home before the bees catch him, the number your program writes to standard output must be -1 instead.

GRADING

For a number of tests, worth a total of 40 points, N will not exceed 60.

EXAMPLES

Sample Input	Sample Output
7 3 TTTTTTT TGGGGGT TGGGGGT MGGGGGD TGGGGGT TGGGGGT THHHHHT	1

After eating honey for one minute, Mecho can take the shortest path directly to the right and he will be home in another two minutes, safe from the bees.

Sample Input	Sample Output
7 3 TTTTTTT TGGGGGT TGGGGGT MGGGGGD TGGGGGT TGGGGGT TGHHGGT	2

After eating honey for two minutes, Mecho can take steps $\rightarrow\uparrow\rightarrow$ during the third minute, then steps $\rightarrow\rightarrow\rightarrow$ during the fourth minute and steps $\downarrow\rightarrow$ during the fifth minute.

Task 2.3 REGIONS (proposed by Long Fan and Richard Peng)

The United Nations Regional Development Agency (UNRDA) has a very well defined organizational structure. It employs a total of N people, each of them coming from one of R geographically distinct regions of the world. The employees are numbered from 1 to N inclusive in order of seniority, with employee number 1, the Chair, being the most senior. The regions are numbered from 1 to R inclusive in no particular order. Every employee except for the Chair has a single supervisor. A supervisor is always more senior than the employees he or she supervises.

We say that an employee A is a manager of employee B if and only if A is Bs supervisor or A is a manager of Bs supervisor. Thus, for example, the Chair is a manager of every other employee. Also, clearly no two employees can be each others managers.

Unfortunately, the United Nations Bureau of Investigations (UNBI) recently received a number of complaints that the UNRDA has an imbalanced organizational structure that favors some regions of the world more than others. In order to investigate the accusations, the UNBI would like to build a computer system that would be given the supervision structure of the UNRDA and would then be able to answer queries of the form: given two different regions r_1 and r_2, how many pairs of employees e_1 and e_2 exist in the agency, such that employee e_1 comes from region r_1, employee e_2 comes from region r_2, and e_1 is a manager of e_2. Every query has two parameters: the regions r_1 and r_2; and its result is a single integer: the number of different pairs e_1 and e_2 that satisfy the above-mentioned conditions.

TASK

Write a program that, given the home regions of all of the agencys employees, as well as data on who is supervised by whom, interactively answers queries as described above.

CONSTRAINTS

$1 \leq N \leq 200\ 000$ – The number of employees
$1 \leq R \leq 25\ 000$ – The number of regions
$1 \leq Q \leq 200\ 000$ – The number of queries your program will have to answer
$1 \leq H_k \leq R$ – The home region of employee k (for $1 \leq k \leq N$)
$1 \leq S_k < k$ – The supervisor of employee k (for $2 \leq k \leq N$)
$1 \leq r_1, r_2 \leq R$ – The regions inquired about in a given query

INPUT

Your program must read from standard input the following data:

• The first line contains the integers N, R and Q, in order, separated by single spaces.

• The next N lines describe the N employees of the agency in order of seniority. The k^{th} of these N lines describes employee number k. The first of these lines (i.e., the one describing the Chair) contains a single integer: the home region H_1 of the Chair. Each of the other $N - 1$ lines contains two integers separated by a single space: employee ks supervisor S_k, and employee ks home region H_k.

INTERACTION

After reading the input data, your program must start alternately reading queries from standard input and writing query results to standard output. The Q queries must be answered one at a time; your program must send the response to the query it has already received before it can receive the next query.

Each query is presented on a single line of standard input and consists of two different integers separated by a single space: the two regions r_1 and r_2.

The response to each query must be a single line on standard output containing a single integer: the number of pairs of UNRDA employees e_1 and e_2, such that e_1s home region is r_1, e_2s home region is r_2 and e_1 is a manager of e_2.

NOTE: The test data will be such that the correct answer to any query given on standard input will always be less than 1 000 000 000.

IMPORTANT NOTE: In order to interact properly with the grader, your program needs to flush standard output after every query response. It also needs to avoid accidentally blocking when reading standard input, as might happen for instance when using `scanf("%d\n")`. Please see the technical info sheet for instructions on how to do this properly.

GRADING

For a number of tests, worth a total of 30 points, R will not exceed 500.

For a number of tests, worth a total of 55 points, no region will have more than 500 employees.

The tests where both of the above conditions hold are worth 15 points.

The tests where at least one of the two conditions holds are worth 70 points.

EXAMPLE

Sample Input	Sample Output
6 3 4	
1	
1 2	
1 3	
2 3	
2 3	
5 1	
1 2	
	1 [flush standard output]
1 3	
	3 [flush standard output]
2 3	
	2 [flush standard output]
3 1	
	1 [flush standard output]

TESTING

If you would like to test your solution through the contest systems test interface, the input file you provide should include both the input data and all queries, as illustrated in the sample input above.

Task 2.4 SALESMAN (proposed by Velin Tzanov)

The traveling salesman has decided that optimally scheduling his trips on land is an intractable computational problem, so he is moving his business to the linear world of the Danube River. He has a very fast boat that can get him from anywhere to anywhere along the river in no time, but unfortunately the boat has terrible fuel consumption. It costs the salesman U dollars for every meter traveled upstream (towards the source of the river) and D dollars for every meter traveled downstream (away from the source of the river).

There are N trade fairs that the salesman would like to visit along the river. Each trade fair is held for one day only. For each trade fair X, the traveling salesman knows its date T_X, measured in the number of days since he purchased his boat. He also knows the fairs location L_X, measured as the distance in meters from the source of the river downstream to the fair, as well as the number of dollars M_X that the salesman is going to gain if he attends this trade fair. He has to start and end his journey at his waterfront home on the river, which is at location S, measured also in meters downstream from the source of the river.

Help the traveling salesman choose which trade fairs to attend (if any) and in what order, so that he may maximize his profit at the end of his travels. The traveling salesmans total profit is defined as the sum of the dollars he gained at the fairs he attended, minus the total sum of dollars he spent traveling up and down the river.

Keep in mind that if trade fair A is held earlier than trade fair B, the salesman can visit them only in this order (i.e., he cannot visit B and then visit A). However, if two fairs are held on the same date, the salesman can visit them both in any order. There is no limit to how many fairs the salesman can visit in a day, but naturally he can't visit the same fair twice and reap the gains twice. He can pass through fairs he has already visited without gaining anything.

TASK

Write a program that, given the date, location and profitability of all fairs, as well as the location of the traveling salesmans home and his costs of traveling, determines the maximum possible profit he can make by the end of his journey.

CONSTRAINTS

$1 \le N \le 500\ 000$ – The number of fairs
$1 \le D \le U \le 10$ – The cost of traveling one meter upstream (U) or downstream (D)
$1 \le S \le 500\ 001$ – The location of the salesmans home
$1 \le T_k \le 500\ 000$ – The day on which fair k is held
$1 \le L_k \le 500\ 001$ – The location of fair k
$1 \le M_k \le 4\ 000$ – The number of dollars the salesman would earn if he attends fair k

INPUT

Your program must read from standard input the following data:

• The first line contains the integers N, U, D and S, in this order, separated by single spaces.

• The next N lines describe the N fairs in no particular order. The k^{th} of these N lines describes the k^{th} fair and contains three integers separated by single spaces: the day of the fair T_k, its location L_k, and its profitability for the salesman M_k.

NOTE: All locations given in the input will be different. That is to say, no two fairs will happen at the same location and no fair will happen at the salesmans home.

OUTPUT

Your program must write to standard output a single line containing a single integer: the maximum profit the salesman can possibly make by the end of his journey.

GRADING

For a number of tests, worth a total of 60 points, no two fairs will be held on the same day. For a number of tests, worth a total of 40 points, none of the numbers in the input will exceed 5 000.

The tests where both of the above conditions hold are worth 15 points.

The tests where at least one of the two conditions holds are worth 85 points.

EXAMPLE

Sample Input	Sample Output
4 5 3 100 2 80 100 20 125 130 10 75 150 5 120 110	50

An optimal schedule would visit fairs 1 and 3 (the ones at locations 80 and 75). The sequence of events and their associated profits and costs would be as follows:

- The salesman travels 20 meters upstream at a cost of 100 dollars. Profit so far: -100
- He attends fair number 1 and earns 100. Profit so far: 0
- He travels 5 meters upstream at a cost of 25. Profit so far: -25
- He attends fair number 3 where he earns 150. Profit so far: 125
- He travels 25 meters downstream to return home at a cost of 75. Profit at the end: 50

TECHNICAL INFO SHEET (day 2)

These pages contain helpful information on how to avoid slow input/output performance with C++ streams (cin / cout), how to use 64-bit data types (variables) and how to properly communicate with the grader on interactive tasks. They also include reference for what options are given to the compilers and what stack limitations are in place.

Slow Input / Output with C++ Streams

When solving tasks with very large amounts of input / output data, you may notice that C++ programs using the cin and cout streams are much slower than equivalent programs that use the scanf and printf functions for input and output processing. Thus, if you are using the cin / cout streams we strongly recommend that you switch to using scanf / printf instead. However, if you still want to use cin / cout, we recommend adding the following line at the beginning of your program:

```
ios::sync_with_stdio(false);
```

and also making sure that you never use `endl`, but use `\n` instead.

Please note, however, that including `ios::sync_with_stdio(false)` breaks the synchrony between cin / cout and scanf / printf, so if you are using this, you should never mix usage of cin and scanf, nor mix cout and printf.

64-bit Data Types

For some tasks you may need to deal with numbers too large to fit in 32 bits. In these cases, you would have to use a 64-bit integer data type, such as `long long` in C/C++ or `int64` in Pascal. Here is some sample code that illustrates the usage of these data types:

C/C++

```
int main(void) {
    long long varname;
    scanf("%lld", &varname);
    // Do something with the varname variable
    printf("%lld\n", varname);
    return 0;
}
```

Pascal

```
var
    varname: Int64;
begin
    read(varname);
    { Do something with the varname variable }
    writeln(varname);
end.
```

Communication with Grader on Interactive Tasks

Whenever you solve an interactive task, you always need to flush the buffer of your output after every new line printed on the output. Here is some code to illustrate how to do this under C, C++ and Pascal:

C or C++ with scanf / printf

```
fflush(stdout);
```

In addition, when using `scanf`, you must avoid reading input in a way that blocks the execution of your program while waiting for white space on standard input. Such blocking might happen if you use `scanf` with a first argument that ends with a space or a new line. In particular, you can safely use `%d` as a `scanf` argument, but you should NOT use `%d ` (with a trailing space) or `%d\n` (with a trailing new line).

C++ with cin / cout

```
cout << flush;
```

Pascal

```
flush(output);
```

Compiler Options

The following commands will be used to compile solutions of batch and interactive tasks (say the task name is abc):

C

```
gcc -o abc abc.c -std=gnu99 -O2 -s -static -lm -x c
```

C++

```
g++ -o abc abc.cpp -O2 -s -static -lm -x c++
```

Pascal

```
fpc -O2 -XS -Sg abc.pas
```

Stack Limitations

Whenever your program is executed through the contest system, the stack size will only be limited by the memory limit for the corresponding task.

Task Overview Sheet (Day 2)

	Garage	Mecho	Regions	Salesman
Type	Batch	Batch	Reactive	Batch
Detailed Feedback	Full	Partial	None	Partial
Time Limit (per test case)	1 second	1 second	8 seconds	3 seconds
Memory Limit (per test case)	32 MB	64 MB	128 MB	128 MB
Points	100	100	100	100

NOTE: C++ programmers should be aware that using C++ streams (cin / cout) may lead to I/O bottlenecks and substantially slower performance. Please see the technical info sheet for ways to avoid this.

SOLUTIONS

0.1 Area

The area of a rectangle with sides A_k and B_k is $A_k \times B_k$, so we simply need to add these products up for each of the rectangles.

As noted in the problem statement, this problem was intended purely to get contestants to try out the 64-bit integer types in their chosen languages. The only thing one must keep in mind is that multiplying two 32-bit values will yield a 32-bit result, even if assigned to a 64-bit variable. The safest thing to do is to use 64-bit variables throughout.

0.2 Hill

First we make an observation: starting at any point in the grid, we can "walk uphill" until reaching a cell with no adjacent higher cells, which is a hill. Furthermore, if we start from a cell with height h inside a region bounded entirely by cells with height less than h (or by the boundaries of the entire grid), then this walk cannot leave this region, and hence we know that a hill must exist inside this region.

We start the algorithm knowing that somewhere in the entire grid we have a hill. The algorithm then repeatedly finds smaller and smaller rectangles known to contain hills. At each step of the algorithm, we will have a rectangle with the following properties:

1. The highest cell seen so far falls inside this rectangle[1].
2. The rectangle is bounded by cells strictly lower than this highest cell, or by the boundaries of the original grid.

Although we do not actually perform an uphill walk, the property noted above guarantees that this rectangle will contain a hill. If the rectangle is 1×1, then it

[1] At the start of the algorithm, we have not seen any cells, but this turns out not to be very important.

consists of just a cell which is a hill, and the problem is finished. Otherwise, we can produce a smaller rectangle as follows.

First, use the laser scanner on every cell in a line that cuts the rectangle in half (either horizontally or vertically, whichever will use the fewest scans). Let H be the highest cell that has been seen so far (including the cells that have just been scanned). Now if H does not lie on the cut, then it falls into one half of the rectangle. This half then satisfies the properties above, and we have successfully reduced the size of the rectangle. If H lies on the cut, then some additional work is required. Scan the cells immediately adjacent to H that have not yet been scanned, and let H' be the new highest cell seen. If $H' = H$ then H is a hill (since we have scanned all its neighbours), and we can immediately terminate. Otherwise, H' does not lie on the cut, and we can proceed to select one half of the rectangle as before.

By repeatedly finding smaller rectangles known to contain a hill, we must eventually find a 1×1 rectangle and the problem is solved. An upper bound on the number of scans required is

$$1002 + 502 + 502 + 252 + 252 + \cdots + 5 + 5 + 3 = 3023$$

Slightly tighter or looser bounds can be obtained depending on exact details of the implementation, but this is not important as full points are awarded as long as the number of scans is at most 3050.

0.3 Museum

To solve this problem, we start by observing that if we have three vases with heights A, B and C, such that either A is odd and C even, or A even and C odd, then no matter what B is these three vases will not violate the condition of the exhibition organisers. This is because $A + C$ must therefore be odd, and so is not divisible by two, meaning that it is impossible for B to be the average of A and C.

We therefore start by arranging the vases such that we place all the even vases first, and all the odd vases second. This gives an arrangement that looks like this:

$$E_1 \quad E_2 \quad \ldots \quad E_p \quad O_1 \quad O_2 \quad \ldots \quad O_q$$

where $E_1, E_2, \ldots, E_p$ are the even heights in some order, and $O_1, O_2, \ldots, O_q$ are the odd heights in some order.

Now consider any heights A, B, C which violate the organisers' requirements. By the observations above, either A and C are both even (in which case B is even, since it appears between A and C and all the even values are grouped together), or A and C are both odd (in which case B is also odd). In other words, we can consider the problems of ordering the even numbers and the odd numbers separately.

Now suppose that $a_1, a_2, \ldots, a_p$ is a permutation of the heights $1, 2, \ldots, p$ which satisfies the organisers' requirements (this is a smaller instance of the problem, so it can be solved by divide-and-conquer). Then simply assigning $E_i = 2a_i$ will satisfy the requirements on the even values. Similarly, given a permutation $b_1, b_2, \ldots, b_q$ of the heights $1, 2, \ldots, q$ which satisfies the requirements, we can assign $O_i = 2b_i - 1$.

Examining the properties of the resulting sequence gives another approach to generating the same solution. We can write all the heights in binary form, and then sort them according to the *reverse* of their binary form. This sorts first on the least significant bit (i.e., on whether they are odd or even), then the next least significant bit and so on. To prove that this solution is valid, note that if B is the average of A and C, then at the least significant bit that A and B first differ, A and C must have the same value for that bit, placing A and C in a separate group from B when sorting on that bit.

1.1 Archery

The proofs below are very verbose and long-winded, but the ideas behind the algorithms are not all that complicated. The steps can be summarized as:

1. A trivial solution (trying all possibilities and simulating the tournament for each one) gives an $O(N^2R)$ algorithm.

2. One can observe that after no more than $2N$ rounds, the tournament becomes cyclical with all positions repeating every N rounds. This allows the trivial algorithm to be sped up to $O(N^3)$.

3. One can optimize the simulation of the tournament (once we have chosen an initial position) from $O(N^2)$ to $O(N)$. This is the most complicated part of the solution. The key here is that we are only interested in our own position at the end, not in everyone else's.

4. Once you have a subroutine that tells you where you will end up given where you start, you can use it with a modified binary search, improving the $O(N^2)$ algorithm to $O(N \log N)$; or alternatively, improving the $O(N^3)$ algorithm to $O(N^2 \log N)$.

5. The last two algorithms above also have slower versions ($O(N^2 \log N)$ and $O(N \log N \log N)$) if you try to solve the problem by also keeping track of other archers' positions, not just your own.

Optimising to $O(N^3)$

A trivial solution is to try all possible starting positions and simulate the tournament for each round, giving complexity of $O(N^2 R)$. We now show how this can be reduced to $O(N^3)$.

Consider the archers numbered from $N + 2$ to $2N$. Let's call them the weak archers.

Theorem 1. *After enough rounds (no more than $2N$) the weak archers will occupy the targets numbered 2 to N, one such archer on each target, and will stay there until the end of the tournament.*

Proof. After $N - 1$ rounds, archer number 1 will be on target 1 and will stay there until the end. From this point on, if we consider the set of N archers consisting of archer number 1 plus the $N - 1$ weak archers (let us call this the *weak+1* set), and if we imagine the targets arranged in a circle (1 to N and then again 1), we have the following scenario:

- When an archer from the weak+1 set competes with an archer outside of weak+1, then the weak+1 archer will stay on the target and the other archer will move.
- When two archers belonging to the weak+1 set compete against each other, one of them will stay and the other will move to the target on his left.

Lemma 1. *Within N rounds after archer number 1 has arrived on target 1, every target will have at least one weak+1 member on it.*

Proof. Suppose the above is not true. We know that once a target is occupied by a weak+1 member, then it will always be occupied by at least one (because weak+1 members never move out of their target unless there is another weak+1 archer to replace them there). Thus if Lemma 1 is false, there must exist a target that is never occupied by a weak+1 member (within the N rounds). Let's call this target A. If A is never occupied by a weak+1 archer, then the target to the left of A (let us call it B) would have at most one such archer within one round and would remain this way. Then within two rounds the target to the left of B would have at most one weak+1 archer, and within three rounds the next target to the left would have at most one such archer. Continuing around the circle, within $N-1$ rounds the target to the right of A would have at most one weak+1 archer. Thus within $N-1$ rounds all targets except A would have at most one weak+1 archer. But since there are N such archers and N targets, this means that A must have at least one weak+1 archer. Since this contradicts our supposition that A remains free of weak+1 archer for N rounds, this proves Lemma 1. $\square$

Now that we know every target has at least one weak+1 archer within $2N$ rounds from the start of the competition, and since we know that once a target has such an archer it never ceases to have at least one, we have proved Theorem 1. $\square$

Now consider all archers that don't belong to weak+1. If we have one weak+1 archer on every target, this also means we also have one non-weak+1 archer on every target. Since under this scenario the weak+1 archers always stay where they are, this means the archers numbered 2 to N+1 will cyclically rotate around the N targets, periodically repeating their positions after every N rounds.

This means that if we replace R by $R' = 2N + (R \mod N)$ we would get an identical answer, since the outcome of the tournament after R rounds would be identical to the outcome after R' rounds (remember that $R \geq 2N$).

The above means that we can easily improve our $O(N^2R)$ algorithm to $O(N^3)$.

Optimising to $O(N^2)$

Currently, when we choose a starting position and we simulate what happens after R' rounds, we do $O(N^2)$ calculations per starting position. We can reduce the complexity of this part to $O(N)$ in the following way.

Observe that there are three types of archers: ones that are better than us, which we'll call the *black archers*; ones that are worse than us, which we'll call the *white archers*; and ourself (a single archer) which we'll denote as the *gray archer*. In order to solve our problem, we need not make any distinctions between archers of the same colour, as it is irrelevant to the final outcome. If two archers of the same colour compete against each other, it does not matter to us which one prevails (i.e., it is not going to impact the gray archer's final position). And we know that whenever two archers of different colours compete against each other, the archer of the darker colour wins.

Now there are three different cases which we'll handle separately.

Case 1 There are no black archers. This means we are the best archer and in this case it is trivial to show that the optimal target to start on is target N.

Case 2 There is at least one black archer, but no more than N. This means that our rank is between 2 and $N+1$, which means we are part of the group of archers that eventually ends up circling the targets periodically. In this case, it is notable that we do not need to simulate the full tournament, but only what happens on target 1. If we know who competes on target 1 every round, then just observing when between rounds $2N$ and $3N$ the gray archer gets to compete against archer number 1 will tell us where the gray archer will finish the tournament (which is all that we are interested in). We will track what happens on target number 1 using the following algorithm:

We assign each archer i a number P_i, which informally speaking indicates the earliest possible round where i might potentially compete on target 1. Initially each archer's P number equals his original target number. Then we simulate each round of the tournament with the following procedure:

1. We determine who is competing on target 1. The first archer there is clearly the winner on target 1 from the previous round (or initially the leftmost archer). We determine his opponent in the following way. We take all archers with P number less than or equal to the number of the current round. The best archer among them will now be competing on target 1 (the proof of why this is correct is further below).

2. We compare these two archers and assign the loser a P value equal to the number of the current (just passed) round plus N. This is the earliest round when we might potentially see this archer back on target 1.

Now let us prove that the above is correct. We will denote with A_j the archer who is competing on target 1 on round j, but who was competing on target 2 on round $j-1$. Every archer i has a value P_i that if he were to win every single contest since getting that P_i value, he would end up being A_{P_i}. Now let's look at the archer selected by our algorithm to be A_j (for some round j). We will denote him by W. Let $S = j - P_W$. If S is zero, this means that W didn't have the opportunity to become A_{j-1} even if he were to win all his contests. Hence, in this cycle W never met A_{j-1} (or any of the earlier A's). Since W never met these archers and since he is better than everybody else who is in the running for A_j, this means that he never lost in this cycle (until he got to target 1 at least), which means he truly is A_j (i.e., our algorithm is correct in this case).

If S is greater than zero, this means that W had the opportunity to become A_{j-1}, but lost it. This means he competed directly with A_{j-1} because the latter was the only candidate for A_{j-1} that was better than W[2]. Now if W competed

[2]This is true because by our algorithm every candidate for A_{j-1} automatically becomes a

with A_{j-1} and if he is better than every other A_j candidate, this means that after their meeting W was always "on the heels" of A_{j-1}: either on the same target, or on the one immediately to the right. This means that when A_{j-1} reached target 1 (which is in round $j-1$), W was on target 2. Since by definition he was better than the other archer on target 2, this means he was indeed the one to reach target 1 on round j.

Now that our algorithm for keeping track of target 1 is proved correct, we can analyze its time complexity. Since we make no distinction between same-coloured archers, we can represent any set of archers by just three numbers: the number of white, gray and black archers in that set. This allows us to execute every step of the algorithm (i.e., every round of simulation) in constant time, because all we have to do is determine the colour of the best archer in a set of candidates and then add to that set only one or two new archers (those whose P value equals the number of the next round). Since we only need to simulate up to round $3N$, and we are not interested in P values above $3N$, we can implement our simulation algorithm in $O(N)$ time and space.

Case 3 There are more than N black archers. This means our number is more than $N+1$, which means that we are one of the archers that eventually ends up standing on the same target indefinitely. We only need to determine which target that is.

We already showed that once archer 1 arrives on target 1, all that the weak+1 archers do is push each other around the targets until they settle on a different target each. Since our number is greater than $N+1$, this means that all white and gray archers belong to the weak set. Thus all we need to do is simulate how the white and gray archers push each other around. We start at target 1 where we know no white/gray archer would be allowed to stay. Then we attempt to count how many white/gray archers end up getting "pushed" around the circle after every target. Initially the white/gray archers pushed from 1 to N would be those that were initially at target 1 (note that our count is still a lower bound; later on we may find out there were even more white/gray archers pushed from target 1). Then we move to target N. We add any white/gray archers that start there to the ones we transferred from target 1 and we leave one of the combined set there (we always leave a white one, if we have one; if not, we leave the gray; if the set is empty, then we obviously do not leave anyone and let the black archers have this

candidate for A_j, except for the actual A_{j-1} — so if W was an A_{j-1} candidate, but did not succeed and is now the best among the A_j candidates, he must have been second to A_{j-1} among the A_{j-1} candidates.

spot). We keep going around the circle from N to $N-1$, to $N-2$, etc. On every target we "pick up" any white/gray archers and we leave one of those we have picked up either earlier or now. Eventually we get to target 1 and if we happen to have any white/gray archers pushed to target 1, we just transfer them to target N and keep going with the same procedure. The second time we return to target 1 we certainly will not have any more white/gray archers to push around, because by Theorem 1 we know that in $2N$ rounds every white or gray archer would have settled on a target. This algorithm clearly runs in linear time and space for the same reasons as the algorithm in Case 2 above. It is also correct because we only move around white/gray archers when necessary (i.e., when they would end up on the same target with another white/gray archer or on target 1) and we make sure that in the end every white/gray archer would have settled somewhere where he can remain undisturbed until the end of the tournament.

The optimization of the tournament simulation from $O(N^2)$ to $O(N)$ described above improves our solution to the whole problem from $O(N^3)$ to $O(N^2)$.

Optimising to $O(N \log N)$

The last optimization that we use to bring the complexity of our algorithm down to $O(N \log N)$ is based on the well-known technique of binary search. The efficient tournament simulation algorithms described above can easily be modified to also tell us how many times the gray archer moves from target 1 to target N (denoted by T). Combining this information with the final position of the gray archer (denoted X) allows us to view the final position on a linear (as opposed to circular) scale. If we describe the outcome of a simulation as being the number $X - N \times T$ we can think of every transfer of the gray archer from one target to another as decrementing the outcome by one. Then if we look at the simulation algorithms described above, we can observe that if the starting position is higher, then the final outcome can never be lower. For example if you choose to start with a larger P value this can never get you further ahead (on the linear scale, not the circular) than if you had chosen a smaller initial P value.

Given this monotonic relationship between the starting position and the final outcome of a tournament, can find the optimal starting position as follows:

1. Measure the outcome of starting on target 1.
2. Measure the outcome of starting on target N.
3. For each multiple of N in this range, use standard binary search to find the smallest possible ending position strictly greater than this multiple (and hence the closest to target 1 for a particular number of wrap-arounds).

4. Of the starting positions found above, pick the best.

Since there are only $O(N)$ rounds being considered, the range to search is $O(N)$ and hence only $O(1)$ binary searches are required. Each such binary search requires $O(\log N)$ starting positions to be tested, giving a time complexity of $O(N \log N)$.

Additional notes

Finally, we should note that the efficient simulation algorithms described above (which ignore distinctions between same-coloured archers and work in $O(N)$ time per simulation) can be implemented in a way that does distinguish between the different black or white archers, using binary heaps or other similar data structures. This would give a final algorithm of complexity $O(N^2 \log N)$ or $O(N \log N \log N)$, depending on whether one also uses the binary search. One can also achieve a time complexity of $O(N^2 \log N)$ or $O(RN \log N)$ by applying the binary search technique without optimizing the simulation.

It is also possible to solve the problem in linear time, but this is very difficult and left as an exercise to the reader. An $O(N \log N)$ solution is sufficient to receive a full score.

1.2 Hiring

Each worker k is described by two numbers: his minimum salary S_k and his qualification Q_k.

Imagine that we already picked a set K of workers we want to hire. How do we compute the total amount of money we need to pay them?

According to the problem statement, the salaries must be proportional to the qualification levels. Hence, there must be some unit salary u such that each employee $k \in K$ will be paid $u \cdot Q_k$ dollars. However, each employee's salary must be at least as large as his minimum salary. Therefore, u must be large enough to guarantee that for each $k \in K$ we have $u \cdot Q_k \geq S_k$.

For more clarity, we can rewrite the last condition as follows: For each $k \in K$ we must have $u \geq S_k/Q_k$. Let us label S_k/Q_k as U_k — the minimum unit cost at which worker k can be employed. We also want to pay as little as possible, hence we want to pick the smallest u that satisfies all the conditions. Therefore we get:

$$u = \max_{k \in K} U_k.$$

Note that this means that the unit salary is determined by a single employee in K — the one with the largest value of U_k.

We just showed that for any set of workers K (therefore also for the optimal set) the unit salary u is equal to the value U_k of one of the workers in K. This means that there are only $O(N)$ possible values of u.

Now imagine that we start constructing the optimal set of workers K by picking the unit salary u. Once we pick u, we know that we may hire only those workers k for which $U_k \leq u$. But how do we determine which of them to hire?

This is easy: if we hire a worker with qualification Q_k, we will have to pay him $u \cdot Q_k$ dollars. In order to maximize the number of workers we can afford (and minimize the cost at which we do so), we clearly want to hire the least-qualified workers.

Hence, we can compute the best solution for a given unit cost u by finding all the workers that we may hire, ordering them by qualification, and then greedily picking them one by one (starting from the least qualified) while we can still afford to pay them.

This gives us an $O(N^2 \log N)$ solution. The solution can easily be improved to $O(N^2)$, as we can sort the workers according to their qualification once in the beginning, and then each possible unit cost u can be tried in $O(N)$.

Finally, we'll show how to improve the above algorithm to $O(N \log N)$. We'll start by ordering all workers according to the value U_k in ascending order, and we label the workers $k_1, k_2, \ldots, k_N$ in this order.

In order to find the optimum set of workers, we'll do exactly the same as in the previous algorithm, only in a more efficient way.

Let $Z(m)$ be the following question: "What is the optimal subset of $\{k_1, \ldots, k_m\}$, given that the unit salary is $U_{k_m} = S_{k_m}/Q_{k_m}$?"

From the discussion above it follows that the optimal solution has to be the answer to a question $Z(m)$ for some m. Hence all we need to do is to answer these N questions.

The inefficient part of the previous solution lies in the fact that for each m we were doing the computation all over again. We can now note that we do not have to do this — we may compute the answer to $Z(m+1)$ from the answer to $Z(m)$, for each m.

Assume that we already know the optimal answer to $Z(m)$ for some m. We will store the workers we picked in a priority queue Q ordered according to their qualification, with more qualified workers having higher priority.

Now we want to add the worker k_{m+1}. His qualification level may make him a better candidate than some of the workers we have already processed. We add him into the priority queue Q. Q now contains all workers we need to consider when looking for the current optimal solution, because if a worker had a qualification too large to be in the optimal solution for m, we will never want to use him again. This holds because the unit cost never decreases and the pool of workers only grows, so the cost of employing a worker together with all available less-qualified workers will only go up.

However, Q may still differ from the optimal answer to $Z(k+1)$, because the cost of paying all the workers in Q might exceed the budget W. There are two reasons for this: first, when adding the worker k_{m+1} the current unit salary u may have increased. And second, even if it stayed the same, we added another worker, and this alone could make the total salary of the chosen workers greater than W.

Hence, we now may need to adjust the set of chosen workers by repeatedly throwing away the most qualified one, until we can afford to pay them all. And this is where the priority queue comes in handy.

To summarize, the 100-point solution we just derived looks as follows: first, order the workers according to the unit salary they enforce. Then, process the workers in the order computed in step 1. Keep the currently optimal set of workers in a priority queue Q, and keep an additional variable T equal to the sum of qualifications of all workers in Q. For each worker k, we first add him into Q (and update T accordingly), and then we throw away the largest elements of Q while we cannot afford to pay them all — that is, while $T \cdot S_{k_m}/Q_{k_m}$ exceeds the amount of money we have.

Once we are finished, we know the numeric parameters of the optimal solution – the optimal number of workers, the minimum cost to hire that many workers, and the number f of the worker for which we found it. To actually construct the solution, it is easiest to start the process once again from the beginning, and stop after processing f workers.

The first step (sorting) can be done in $O(N \log N)$.

In the second step (finding the optimal number of workers and the cost of hiring them), for each worker we insert his qualification into Q once, and we remove it from Q at most once. Hence there are at most $2N$ operations with the priority queue, and each of those can be done in $O(\log N)$ (e.g., if the priority queue is implemented as a binary heap).

The third step (constructing one optimal set of workers) takes at most as long as the second step.

Therefore the total time complexity of this solution is $O(N \log N)$.

Alternative solution Instead of iterating m upwards, it is also possible to iterate it downwards. Suppose that P is the optimal subset of $\{k_1, \ldots, k_m\}$ with

$u = U_{k_m}$, and we wish to modify P to find the optimal subset of $\{k_1, \ldots, k_{m-1}\}$ with $u = U_{k_{m-1}}$. Firstly, we must remove k_m from Q if it is currently present. By potentially having reduced u and/or removed a worker, we may have freed up more money to hire workers. But which workers should we hire?

Clearly we cannot hire any workers that we are already employing. Also, the only reason we ever remove a worker k from P is because u fell below U_k, and since u only decreases that worker can never be hired again. Hence, we can maintain a simple queue of workers, ordered by qualification, and just hire the next available worker until there is not enough money to do so. It is also necessary to remove workers from this queue when u decreases, but this can be achieved by flagging workers as unemployable and skipping over them.

Each worker can be added to the optimal set at most once, and removed from the optimal set at most once. Each of these steps requires only constant time, so the core of this algorithm requires $O(N)$ time. However, the initial sorting still requires $O(N \log N)$ time.

1.3 POI

This problem is intended as a straight-forward implementation exercise. After the data is loaded from the file, a first pass over it can be used to count the number of people not solving each task (and hence the number of points assigned to each task). A second pass then suffices to determine, for each contestant, the number of tasks solved and the score.

It is not necessary to completely determine the final ranking: Philip's rank is simply the number of contestants that appear before him in the ranking, plus one. This can be determined by comparing each contestant to Philip. A contestant C will appear ahead of Philip in the ranking if and only if

- C has a higher score than Philip; or
- C has the same score as Philip, but has solved more tasks; or
- C has the same score as Philip and has solved the same number of tasks, but has a lower ID.

1.4 Raisins

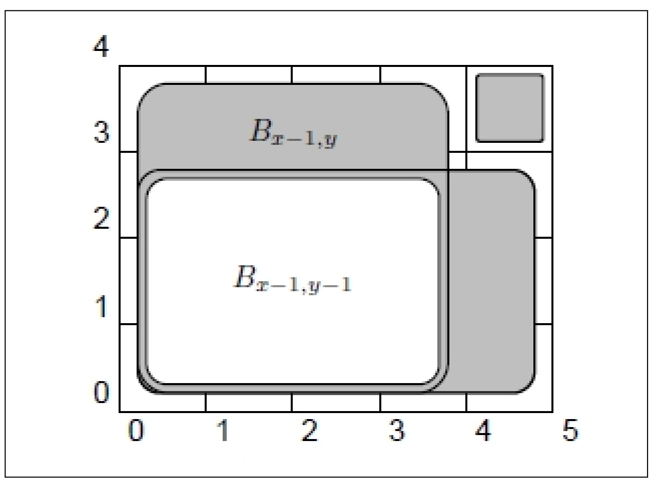
At any moment during the cutting, we have a set of independent sub-problems — blocks of chocolate. If we find the optimal solution for each of the blocks, together we get the optimal solution for the whole chocolate. This clearly hints at a dynamic programming solution.

Each sub-problem we may encounter corresponds to a rectangular part of the chocolate, and it can be described by four coordinates: specifically, two x and two y coordinates — the coordinates of its upper left and lower right corner. Hence we have $O(N^4)$ sub-problems to solve.

Now to solve a given sub-problem, we have to try all possible cuts. There are $O(N)$ possible cuts to try — at most $N-1$ horizontal and $N-1$ vertical ones. Each possible cut gives us two new, smaller sub-problems we solve recursively. Obviously, the recursion stops as soon as we reach a 1×1 block.

Assume that someone has given us a function $S(x_1, y_1, x_2, y_2)$ that returns the number of raisins in the rectangle given by coordinates (x_1, y_1) and (x_2, y_2) in constant time.

Using this function we can solve the entire problem in $O(N^5)$. We will use recursion with memoization. Given any of the $O(N^4)$ sub-problems, first check the memoization table to see whether we have computed it already. If yes, simply return the previously computed value. Otherwise, proceed as follows: The cost of the first cut is $S(x_1, y_1, x_2, y_2)$, which we have supposed can be computed in $O(1)$ time. For each possible placement of the first cut, recursively determine the cost of the remaining cuts in each sub-problem, and pick the optimal choice, storing the answer in the memoization table.



We are only missing one piece of the puzzle: the function S. All possible values can easily be precomputed in $O(N^4)$ and stored in an array.

Alternatively, we can use two-dimensional prefix sums: let A be the input array, and let $B_{x,y} = \sum_{i<x} \sum_{j<y} A_{i,j}$. The values B are called two-dimensional prefix sums. They can be computed using the formula

$$\forall x, y > 0 : \; B_{x,y} = B_{x-1,y} + B_{x,y-1} - B_{x-1,y-1} + A_{x-1,y-1}.$$

Having the two-dimensional prefix sums, we can compute the sum in any rectangle, using a similar formula. The sum in the rectangle with corners (x_1, y_1) and (x_2, y_2) is

$$S(x_1, y_1, x_2, y_2) = B_{x_2,y_2} - B_{x_1,y_2} - B_{x_2,y_1} + B_{x_1,y_1}.$$

2.1 Garage

The problem is essentially a straight-forward simulation, but the data structures required are not completely trivial. A simple implementation that does not require any kind of data structure beyond a fixed-size array will keep track of:

- for each car, its state (absent, in the queue, or parked), its parking space (if parked), and its arrival time (if in the queue);
- for each parking space, whether there is a car parked there.

Now one can process the input events one at a time. When a car arrives, loop over all parking spaces to find the first empty one. If one is found, park the car there. Otherwise, the car will have to go into the queue — so record its arrival time.

When a car leaves the garage, it will be replaced by the car at the front of the queue (if any). Loop over all cars to find the car that arrived earliest and is still in the queue. If one is found, park it in the parking space that has just been freed up, and mark it as no longer in the queue.

This solution runs in $O(M^2 + MN)$ time. This can be improved: keeping the queue in a separate array reduces this to $O(MN)$, and also keeping the available parking spaces in a binary heap reduces it to $O(M \log N)$. However, these optimisations are not necessary to receive a full score.

2.2 Mecho

Solution 1

Firstly, working with fractional amounts of time is tricky, so we will measure time in units of $\frac{1}{S}$ seconds — let's call them *ticks*. Bees take S ticks to move from one cell to the next, while Mecho takes one tick.

Let us try to solve an easier problem first. Suppose we know when Mecho leaves the honey: can he get home safely? If we can solve this problem, then we can use it inside a binary search to find the last moment at which he can leave.

Mecho's moves will depend on the bees, but the bees' moves are fixed, so let us deal with the bees first. A standard breadth-first search will tell us when the bees reach each grassy cell (this just means simulating the spread of the bees over time).

Next, we can perform a similar breadth-first search for Mecho to answer the question "How soon (if at all) can Mecho reach each cell?" This can be implemented almost exactly as for the bees, except that one must exclude any steps that would have Mecho move to a cell where he would immediately be caught.

These breadth-first searches can each be implemented in $O(N^2)$ time. The problem statement guarantees that Mecho will eventually be caught if he stays with the honey, it takes $O(N^2)$ seconds for the bees to cover all the cells they can reach, and we are only interested in integer numbers of seconds in the binary search. Thus, the range of values explored by the binary search is $O(N^2)$ and hence the time complexity of this solution is $O(N^2 \log N)$.

Solution 2

Instead of using a binary search, we can use a more complicated method to directly determine the optimal time to leave any cell. The bees are processed as in the first solution. However, instead of working from the honey towards Mecho's home, we start from his home. Since he is safe in his home, there is no limit on how late he can arrive there.

Now suppose we know that for some cell Y, Mecho must leave no later than t ticks (from the time the alarm went off) and still make it home safely. If X is a neighbouring cell of Y, what is the latest time Mecho can leave cell X to safely make it home via Y? Clearly $t - 1$ is an upper bound, otherwise he will reach Y too late. However, he must also leave Y before the bees get there. The latest he can stay will be just the minimum of the two constraints.

One can now do a priority-first search: simulate backwards in time, keeping track of the latest time to leave each cell (keeping in mind that X has other neighbours, and it might be better to leave via those than via Y).

The time complexity of this solution depends on the priority queue used to order cells by leaving time. A binary heap gives an $O(N^2 \log N)$ implementation, and this is sufficient for a full score. However, it can be shown that the number of different priorities that are in the priority queue at any one time is $O(1)$, which makes an $O(N^2)$ solution possible.

2.3 Regions

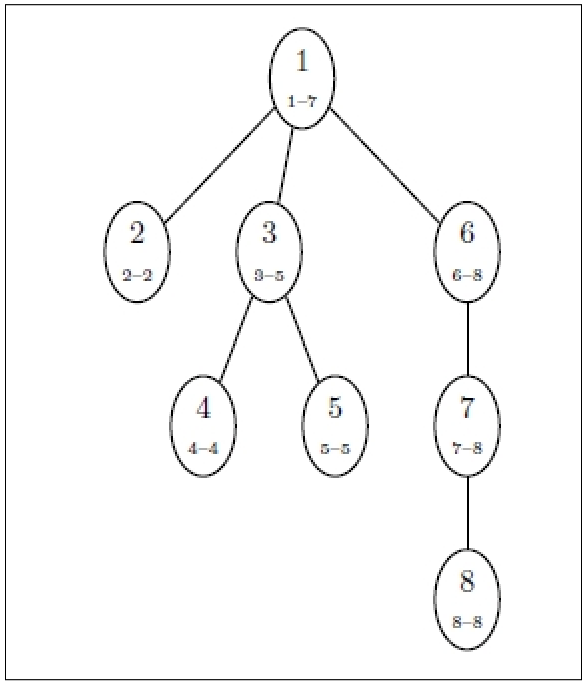
Although the employees are already assigned numbers in the input, the numbers can be reassigned in a way that makes them more useful. The supervisor relationships clearly organise the employees into a tree. Assign the new employee numbers in a pre-order walk of the tree[3]. Figure shows an example of such a numbering.

A useful property of this numbering is that all the employees in a sub-tree have sequential numbers. For a given employee e, let $[e]$ be the range of employee numbers managed by e. Notice that for a region, we can construct an ordered array of all the interval end-points for that region, and a list of all employees in that region. This can be done during the assignment of numbers in linear time.

Now let us consider how to answer queries (r_1, r_2). Let the sizes of the regions be S_1 and S_2 respectively. Given this data structure, a natural solution is to consider every pair of employees (e_1, e_2) from these regions and check whether e_2 lies in the interval $[e_1]$. However, this will take $O(S_1 S_2)$ time per query, which we can improve upon.

The interval end-points for region r_1 divide the integers into contiguous *blocks*. All employees in the same block have the same managers from r_1, and we can precompute the number of such managers for each such block. This gives us a faster way to answer queries. Rather than comparing every employee in r_2 with every block for r_1, we can observe that both are ordered by employee ID. Thus, one can maintain an index into each list, and in each step advance whichever index is lagging behind the other. Since each index traverses a list once, this takes $O(S_1 + S_2)$ time.

[3] A *pre-order walk* of a tree first processes the root of that tree, then recursively processes each sub-tree in turn.



An example of numbering employees by a pre-order walk. The bottom numbers indicate the range of employee numbers in each sub-tree.

Using just this query mechanism can still take $O(NQ)$ time, because all the queries might involve large regions. However, it is sufficient to earn the points for the tests where no region has more than 500 employees.

Precomputing queries

In the query algorithm above, it is also possible to replace the list of employees in r_2 with the entire list of employees, and thus compute the answer to all queries for a particular r_1. This still requires only a single pass over the blocks for r_1, so it takes $O(N)$ time to produce all the answers for a particular r_1. Similarly, one can iterate over all interval end-points while fixing r_2, giving all answers for a particular r_2.

This allows all possible queries to be pre-computed in $O(RN)$ time and $O(R^2)$ memory. This is sufficient to earn the points for the tests where $R \leq 500$.

This algorithm is too slow and uses too much memory to solve all the tests. However, it is not necessary to precompute all answers, just the most expensive ones. We will precompute the answers involving regions with size at least c. There are obviously at most N/c such regions, so this will take $O(N^2/c)$ time and $O(RN/c)$ memory. The remaining queries involve only small regions, so they can be answered in $O(Qc)$ time. Choosing $c = \sqrt{N}$ gives $O(N\sqrt{N} + Q\sqrt{N})$ time and $O(R\sqrt{N})$ memory, which is sufficient for a full score.

Caching queries

As an alternative to precomputation, one can cache the results of all queries, and take the answer from the cache if the same query is made again. Let Q' be the number of unique queries. The cost of maintaining the query cache depends on the data structure used; a balanced binary tree gives $O(Q \log N)$ overhead for this.

Combining the cache with the $O(S_1 + S_2)$ algorithm is sufficient to achieve the points for tests that have either no more than 500 employees per region (because this is the case even without the cache), as well as the cases with no more than 500 regions (since the total cost of all distinct queries together is $O(RN)$).

To achieve a full score with a cache rather than precomputation, one must use a better method for answering queries. Suppose we have a block in r_1, and wish to find all matching employees from r_2. While we have previously relied on a linear walk over the employees from r_2, we can instead use a binary search to find the start and end of the range in $O(\log S_2)$ time. This allows the entire query to be answered in $O(S_1 \log S_2)$ time. A similar transformation (binary searching the blocks for each employee in r_2) gives $O(S_2 \log S_1)$ time for each query.

Now when answering each query, choose the best out of the $O(S_1 \log S_2)$, $O(S_2 \log S_1)$ and $O(S_1 + S_2)$ query mechanisms. To establish an upper bound on run-time, we will make assumptions about which method is chosen to answer particular types of queries.

Again, divide the problem into large regions with at least c employees and the rest. For queries involving one of the large regions, use the $O(A \log B)$ algorithm (where A and B are respectively the smaller and larger of S_1 and S_2). The caching of queries ensures that this contributes no more than $O(N^2 \log N/c)$ time. For the remaining queries, use an $O(S_1 + S_2)$ algorithm. The smaller regions have at most c employees, so this contributes $O(Qc)$ time.

The optimal value of c occurs when the two parts account for equal time. Solving for this optimal c gives a bound of $O(N\sqrt{Q' \log N})$ for answering non-duplicate queries; combined with the cost for the query cache, this gives an algorithm with time complexity $O(N\sqrt{Q' \log N} + Q \log N)$ and memory complexity $O(N + Q')$.

The time bound is marginally worse than for the first solution, but in practical terms this solution runs at about the same speed and uses significantly less memory.

2.4 Salesman

We'll start by considering only the case where no two fairs occur on the same day. Later we'll show how to modify our algorithm to incorporate fairs that occur on the same day.

The first polynomial solution

First we'll describe a fairly standard dynamic programming algorithm. We order the fairs according to the day when they take place. For each fair i we will compute the best profit P_i we can achieve immediately after visiting this fair.

To avoid special cases, we'll add dummy fairs 0 and $N + 1$ which both take place at the salesman's home, fair 0 being the first and fair $N + 1$ the last of all fairs. We can immediately tell that $P_0 = 0$ and that P_{N+1} is the answer we are supposed to compute.

The values P_1 to P_{N+1} can all be computed in order, using the same observation: we have to arrive from some fair, and we may pick which one it is.

Let $cost(x, y)$ be the cost of travelling from point x to point y on the river. If $x \le y$, we have $cost(x, y) = (y - x)D$, otherwise we have $cost(x, y) = (x - y)U$.

We can then write:

$$\forall i \in \{1, \dots, N+1\} : \qquad P_x = \max_{0 \le j < i} (P_j - cost(L_j, L_i)) + M_i$$

(Explanation: To compute P_i we pick the number j of the fair we visited immediately before fair i. Immediately after fair j the best profit we could have was P_j. We then have to travel to the location of the current fair, which costs us $cost(L_j, L_i)$, and finally we visit fair i for a profit M_i. To obtain the largest possible P_i we take the maximum over all possible choices of j.)

The time complexity of this algorithm is $O(N^2)$, which is sufficient to solve the cases where all the input values are at most 5,000.

An improved solution

We will now improve the previous algorithm. Note that the profit M_i from visiting fair i is the same for all choices of j. Thus, the optimal choice of j depends on the profits $P_0, \ldots, P_{i-1}$, the locations $L_0, \ldots, L_{i-1}$, and the location L_i of the current fair.

We can divide the fairs 0 to $i-1$ into two groups: those upstream of L_i, and those downstream. We can now divide our problem "find the optimal j" into two subparts: "find the optimal choice for the previous fair upstream" and "find the optimal choice for the previous fair downstream".

Consider locating the optimal previous fair upstream of L_i. If we were to change the value L_i (in such a way that it does not change which other fairs are upstream of fair i), can it influence our choice? No, it can not. If we, for example, increase L_i by Δ, this means that for *each* of the upstream fairs the cost of travelling to fair i increases by the same amount: $D\Delta$. Hence the optimal choice would remain the same.

We will now show a relatively simple data structure that will allow us to locate the optimal previous fair upstream of fair i in $O(\log N)$ time.

The data structure is commonly known as an interval tree. We can assign the fairs new labels according to their unique positions on the river. More precisely, let l_f be the number of fairs that are upstream of fair f (including those that occur after fair f).

Our interval tree is a complete binary tree with k levels, where k is the smallest integer such that $2^{k-1} \geq N+2$. Note that $k = O(\log N)$.

Leaves in this binary tree correspond to the fairs, and the order in which fairs are assigned to leaves is given by the values l_i. That is, the leftmost leaf is the fair closest to the river source, the second leaf is the second-closest fair, and so on.

Now note that each node in our tree corresponds to an interval of fairs — hence the name "interval tree". In each node of the interval tree we will store the answer to the following question: "Let S be the set of fairs that correspond to leaves in this subtree and were already processed. Supposing that I'm downstream from each of them, which one is the optimal choice?"

Given this information, we can easily determine the optimal choice for the next fair i in $O(\log N)$. And it is also easy to update the information in the tree after fair i was processed; this too can be done in $O(\log N)$.

In our solution we will, of course, have two interval trees: one for the direction upstream and one for the direction downstream. For each fair i, we first make two queries to determine the best predecessor upstream and downstream, then we pick the better of those two choices, compute P_i, and finally we update both interval trees.

Hence we process each fair in $O(\log N)$, leading to the total time complexity $O(N \log N)$.

Another equally good solution

In this section we will show another solution with the same complexity, which uses an "ordered set" data structure only, and can easily be implemented in C++ using the `set` class.

As before, we will process the fairs one by one, ordered by the day on which they occur. Imagine a situation after we have processed some fairs. Let a and b be two fairs that we have already processed. We say that a is *covered by* b if $P_a \leq P_b - cost(L_b, L_a)$.

In human words, a is covered by b if the strategy "visit fair b last and then move to the location of fair a" is at least as good as the strategy "visit fair a last".

Once a fair a is covered by some other fair b, this fair will never be an optimal predecessor for any later fair. Fair b will always (regardless of the location of the later fair) be at least as good a choice as a.

On the other hand, if a fair is currently not covered by any other fair, there are some locations on the river for which b would be the optimal predecessor — at least the location L_b and its immediate surroundings. We will call such fairs *active*.

In our solution we will maintain the set of currently active fairs, ordered by their position on the river. We will use an "ordered set" data structure, most commonly implemented as a balanced binary tree.

It can easily be shown that for each active fair f there is an interval of the river where f is the optimal choice. These intervals are obviously disjoint (except possibly for their endpoints), and together they cover the entire river. And as the interval for f contains f, the intervals are in the same order as their corresponding active fairs.

Hence whenever we are going to process a new fair i, we only have to locate the closest active fairs upstream and downstream of i — one of these two must be the optimal choice.

After we process the fair i and compute P_i, we have to update the set of active fairs. Clearly, i is now active, as we computed P_i by taking the best way of getting to L_i, and then added a positive profit M_i. We add it into the set of active fairs. But we are not done yet — i might now cover some of the previously active fairs.

But these are easy to find: if neither of the immediate neigbours of i (in the set of active fairs) is covered by i, we are obviously done. If some of them are covered by i, erase them from the set and repeat the check again.

In this solution, each fair is inserted into the set of active fairs once, and is erased from the set at most once. In addition, when processing each fair we make one query to find the closest two active fairs. Each of these operations takes $O(\log N)$, hence the total time complexity is $O(N \log N)$.

Multiple fairs on the same day

First of all, note that we cannot process fairs that are on the same day one by one — because we must allow the salesman to visit them in a different order than the one we picked.

There may be many ways in which to visit the fairs on a given day. However, we don't need to consider all of them, just some subset that surely contains the optimal solution.

Suppose that we already picked some order in which to visit the fairs on a given day. Let u and d be the fairs furthest upstream and downstream we visit. We can then, obviously, visit all fairs between u and v as well, as we'll surely be travelling through their locations. And clearly to visit all of these fairs, it's enough to travel first to u and then from u to v, or vice versa. We will only consider such paths.

We will process each day in two phases. In the first phase, we process each fair i separately, as if it were the only fair that day, and we determine a preliminary value P_i — the best profit we can have after coming to fair i from some fair on a previous day.

In the second phase we will take travelling upstream or downstream into account. We will consider each direction separately. When processing a direction, we'll process the fairs in order, and for each of them we'll determine whether it is more profitable to start at this fair (i.e., use the value computed in the previous step) or to start sooner (i.e., use the optimal value computed for the previous fair in this step, subtract the cost of travel from that fair to this one, and add the profit from this fair).

For each fair i, the actual value P_i is then equal to the larger of the two values we get for travelling upstream and downstream.

Finally, we need to update the set of active fairs. When using an interval tree data structure as in Section , this is accomplished simply by adding each fair to the upstream and downstream interval trees. When using an ordered set as in Section , one must take a little more care, as not all of the fairs that we have just processed will be active. This is easily catered for by modifying our update process — before inserting a new active fair, we check that the fair is actually active by examining its potential neighbours in the data structure. If either of the neighbouring fairs covers the one being added, then it is not active, and so should not be added to the active fairs set. With this modification, we can update the entire data structure by sequentially attempting to add each fair (in any order).

Clearly, the additional time needed to process the second phase on any day is linear in the number of fairs that day, assuming we already have them sorted according to their location (which is easily accomplished by adding this as a tie-breaker to the comparison function used to sort all fairs in the beginning). Furthermore, the update steps for the interval tree and ordered set both take $O(\log N)$ time. Therefore this extra step does not change the total time complexity of our algorithm: it is still $O(N \log N)$.

Problem Proposers:

Day 0

0.2 Hill - Iskren Chernev
0.3 Museum - Boyko Bantchev

Day 1

1.1 Archery - Velin Tzanov
1.2 Hiring - Velin Tzanov
1.3 POI - Carl Hultquist
1.4 Raisins - Emil Kelevedjiev

Day 2

2.1 Garage - Carl Hultquist
2.2 Mecho - Carl Hultquist
2.3 Regions - Long Fan and Richard Peng
2.4 Salesman - Velin Tzanov

Reserve Problems

Bruce Merry, Mihai Patrascu, Kentaro Imajo