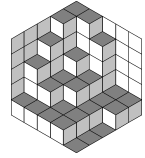




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Day 1

- 1] Let p be a prime number, $p \neq 3$, and integers a, b such that $p|a+b$ and $p^2|a^3+b^3$. Prove that $p^2|a+b$ or $p^3|a^3+b^3$.
- 2] Prove that for every $n \in \mathbb{N}^*$ exists a multiple of n , having sum of digits equal to n .
- 3] Let ABC be an acute-angled triangle. We consider the equilateral triangle $A'UV$, where $A' \in (BC)$, $U \in (AC)$ and $V \in (AB)$ such that $UV \parallel BC$. We define the points B', C' in the same way. Prove that AA', BB' and CC' are concurrent.
- 4] Let ABC be a triangle, and D the midpoint of the side BC . On the sides AB and AC we consider the points M and N , respectively, both different from the midpoints of the sides, such that

$$AM^2 + AN^2 = BM^2 + CN^2 \text{ and } \angle MDN = \angle BAC.$$

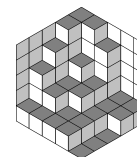
Prove that $\angle BAC = 90^\circ$.

- 5] Let n be an integer, $n \geq 2$, and the integers a_1, a_2, \dots, a_n , such that $0 < a_k \leq k$, for all $k = 1, 2, \dots, n$. Knowing that the number $a_1 + a_2 + \dots + a_n$ is even, prove that there exists a choosing of the signs $+$, respectively $-$, such that

$$a_1 \pm a_2 \pm \dots \pm a_n = 0.$$



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Day 2

- 1] Consider the acute-angled triangle ABC , altitude AD and point E - intersection of BC with diameter from A of circumcircle. Let M, N be symmetric points of D with respect to the lines AC and AB respectively. Prove that $\angle EMC = \angle BNE$.
- 2] In a sequence of natural numbers a_1, a_2, \dots, a_n every number a_k represents sum of the multiples of the k from sequence. Find all possible values for n .
- 3] Let n be a positive integer and let a_1, a_2, \dots, a_n be positive real numbers such that:

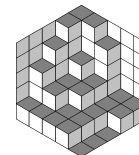
$$\sum_{i=1}^n a_i = \sum_{i=1}^n \frac{1}{a_i^2}.$$

Prove that for every $i = 1, 2, \dots, n$ we can find i numbers with sum at least i .

- 4] Let a, b be real nonzero numbers, such that number $[an + b]$ is an even integer for every $n \in \mathbb{N}$. Prove that a is an even integer.



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Day 3

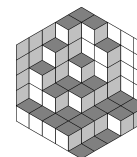
- 1 From numbers $1, 2, 3, \dots, 37$ we randomly choose 10 numbers. Prove that among these exist four distinct numbers, such that sum of two of them equals to the sum of other two.
- 2 Let a, b, c be positive reals with $ab + bc + ca = 3$. Prove that:

$$\frac{1}{1 + a^2(b + c)} + \frac{1}{1 + b^2(a + c)} + \frac{1}{1 + c^2(b + a)} \leq \frac{1}{abc}.$$

- 3 Solve in prime numbers $2p^q - q^p = 7$.
- 4 Let d be a line and points M, N on the d . Circles $\alpha, \beta, \gamma, \delta$ with centers A, B, C, D are tangent to d , circles α, β are externally tangent at M , and circles γ, δ are externally tangent at N . Points A, C are situated in the same half-plane, determined by d . Prove that if exists an circle, which is tangent to the circles $\alpha, \beta, \gamma, \delta$ and contains them in its interior, then lines AC, BD, MN are concurrent or parallel.



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Day 4

- 1] Let $ABCD$ be a convex quadrilateral with opposite side not parallel. The line through A parallel to BD intersect line CD in F , but parallel through D to AC intersect line AB at E . Denote by M, N, P, Q midpoints of the segments AC, BD, AF, DE . Prove that lines MN, PQ and AD are concurrent.
- 2] Let m, n be two natural nonzero numbers and sets $A = \{1, 2, \dots, n\}, B = \{1, 2, \dots, m\}$. We say that subset S of Cartesian product $A \times B$ has property (j) if $(a - x)(b - y) \geq 0$ for each pairs $(a, b), (x, y) \in S$. Prove that every set S with property (j) has at most $m + n - 1$ elements.
- 3] Find all pairs (m, n) of integer numbers $m, n > 1$ with property that $mn - 1 \mid n^3 - 1$.
- 4] Determine the maximum possible real value of the number k , such that

$$(a + b + c)\left(\frac{1}{a + b} + \frac{1}{c + b} + \frac{1}{a + c} - k\right) \geq k$$

for all real numbers $a, b, c \geq 0$ with $a + b + c = ab + bc + ca$.