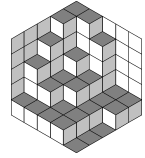




Romania
Junior Balkan Team Selection Tests
Iasi and Bucharest 2006



Day 3 - 16 May 2006

- 1] Let $ABCD$ be a cyclic quadrilateral of area 8. If there exists a point O in the plane of the quadrilateral such that $OA+OB+OC+OD = 8$, prove that $ABCD$ is an isosceles trapezoid.
- 2] Prove that for all positive real numbers a, b, c the following inequality holds

$$\left(\frac{a}{b} + \frac{b}{c} + \frac{c}{a}\right)^2 \geq \frac{3}{2} \cdot \left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right).$$

- 3] Find all real numbers a and b such that

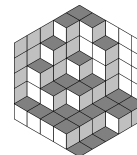
$$2(a^2 + 1)(b^2 + 1) = (a + 1)(b + 1)(ab + 1).$$

Valentin Vornicu

- 4] Prove that the set of real numbers can be partitioned in (disjoint) sets of two elements each.



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Day 4 - 19 May 2006

- 1] Let $A = \{1, 2, \dots, 2006\}$. Find the maximal number of subsets of A that can be chosen such that the intersection of any 2 such distinct subsets has 2004 elements.
- 2] Let ABC be a triangle and A_1, B_1, C_1 the midpoints of the sides BC, CA and AB respectively. Prove that if M is a point in the plane of the triangle such that

$$\frac{MA}{MA_1} = \frac{MB}{MB_1} = \frac{MC}{MC_1} = 2,$$

then M is the centroid of the triangle.

- 3] Let $a, b, c > 0$ be real numbers with sum 1. Prove that

$$\frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{a} \geq 3(a^2 + b^2 + c^2).$$

- 4] The set of positive integers is partitionated in subsets with infinite elements each. The question (in each of the following cases) is if there exists a subset in the partition such that any positive integer has a multiple in this subset.
- a) Prove that if the number of subsets in the partition is finite the answer is yes.
- b) Prove that if the number of subsets in the partition is infinite, then the answer can be no (for a certain partition).