Romania

Junior Balkan Team Selection Tests

Iasi and Bucharest 2006

Day 3 - *16 May 2006*

1 Let $ABCD$ be a cyclic quadrilateral of area 8. If there exists a point O in the plane of the quadrilateral such that $OA+OB+OC+OD = 8$, prove that $ABCD$ is an isosceles trapezoid.

2 Prove that for all positive real numbers a, b, c the following inequality holds

$$\left(\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\right)^2 \geq \frac{3}{2}\cdot\left(\frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b}\right).$$

3 Find all real numbers a and b such that

$$2(a^2+1)(b^2+1) = (a+1)(b+1)(ab+1).$$

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4 Prove that the set of real numbers can be partitioned in (disjoint) sets of two elements each.

Day 4 - *19 May 2006*

1. Let $A = \{1, 2, \ldots, 2006\}$. Find the maximal number of subsets of A that can be chosen such that the intersection of any 2 such distinct subsets has 2004 elements.

2. Let ABC be a triangle and A_1, B_1, C_1 the midpoints of the sides BC, CA and AB respectively. Prove that if M is a point in the plane of the triangle such that
$$\frac{MA}{MA_1} = \frac{MB}{MB_1} = \frac{MC}{MC_1} = 2,$$
then M is the centroid of the triangle.

3. Let $a, b, c > 0$ be real numbers with sum 1. Prove that
$$\frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{a} \geq 3(a^2 + b^2 + c^2).$$

4. The set of positive integers is partitionated in subsets with infinite elements each. The question (in each of the following cases) is if there exists a subset in the partition such that any positive integer has a multiple in this subset.

   a) Prove that if the number of subsets in the partition is finite the answer is yes.

   b) Prove that if the number of subsets in the partition is infinite, then the answer can be no (for a certain partition).