Day 1 - *31 March 2005*

1. Let $\mathcal{C}_1(O_1)$ and $\mathcal{C}_2(O_2)$ be two circles which intersect in the points A and B. The tangent in A at $\mathcal{C}_2$ intersects the circle $\mathcal{C}_1$ in C, and the tangent in A at $\mathcal{C}_1$ intersects $\mathcal{C}_2$ in D. A ray starting from A and lying inside the $\angle CAD$ intersects the circles $\mathcal{C}_1$, $\mathcal{C}_2$ in the points M and N respectively, and the circumcircle of $\triangle ACD$ in P.

   Prove that $AM = NP$.

2. Find the largest positive integer $n > 10$ such that the residue of n when divided by each perfect square between 2 and $\dfrac{n}{2}$ is an odd number.

3. In a country 6 cities are connected two by two with round-trip air routes operated by exactly one of the two air companies in that country.

   Prove that there exist 4 cities A, B, C and D such that each of the routes $A \leftrightarrow B$, $B \leftrightarrow C$, $C \leftrightarrow D$ and $D \leftrightarrow A$ are operated by the same company.

   Dan Schwartz

Day 2 - *01 April 2005*

4 Let a, b, c be positive numbers such that $a+b+c \geq \frac{1}{a}+\frac{1}{b}+\frac{1}{c}$. Prove that

$$a+b+c \geq \frac{3}{abc}.$$

5 On the sides AD and BC of a rhombus $ABCD$ we consider the points M and N respectively. The line MC intersects the segment BD in the point T, and the line MN intersects the segment BD in the point U. We denote by Q the intersection between the line CU and the side AB and with P the intersection point between the line QT and the side CD.

Prove that the triangles QCP and MCN have the same area.

6 Let ABC be an equilateral triangle and M be a point inside the triangle. We denote by A', B', C' the projections of the point M on the sides BC, CA and AB respectively. Prove that the lines AA', BB' and CC' are concurrent if and only if M belongs to an altitude of the triangle.

Day 3 - *19 April 2005*

7 A phone company starts a new type of service. A new customer can choose k phone numbers in this network which are call-free, whether that number is calling or is being called. A group of n students want to use the service.

(a) If $n \geq 2k+2$, show that there exist 2 students who will be charged when speaking.

(b) It $n = 2k+1$, show that there is a way to arrange the free calls so that everybody can speak free to anybody else in the group.

Valentin Vornicu

8 Let a, b, c be three positive reals such that $(a+b)(b+c)(c+a) = 1$. Prove that the following inequality holds:

$$ab + bc + ca \leq \frac{3}{4}.$$

Cezar Lupu

9 Let ABC be a triangle with $BC > CA > AB$ and let G be the centroid of the triangle. Prove that

$$\angle GCA + \angle GBC < \angle BAC < \angle GAC + \angle GBA.$$

Dinu Serbanescu

10 Let $k, r \in \mathbb{N}$ and let $x \in (0,1)$ be a rational number given in decimal representation

$$x = 0.a_1a_2a_3a_4\ldots.$$

Show that if the decimals $a_k, a_{k+r}, a_{k+2r}, \ldots$ are canceled, the new number obtained is still rational.

Dan Schwarz

Day 4 - *23 May 2005*

11 Three circles $\mathcal{C}_1(O_1)$, $\mathcal{C}_2(O_2)$ and $\mathcal{C}_3(O_3)$ share a common point and meet again pairwise at the points A, B and C. Show that if the points A, B, C are collinear then the points Q, O_1, O_2 and O_3 lie on the same circle.

12 Find all positive integers n and p if p is prime and

$$n^8 - p^5 = n^2 + p^2.$$

Adrian Stoica

13 The positive integers from 1 to n^2 are placed arbitrarily on the n^2 squares of a $n \times n$ chessboard. Two squares are called *adjacent* if they have a common side. Show that two opposite corner squares can be joined by a path of $2n-1$ adjacent squares so that the sum of the numbers placed on them is at least $\left\lfloor \frac{n^3}{2} \right\rfloor + n^2 - n + 1$.

Radu Gologan

14 Let a, b, c be three positive real numbers with $a + b + c = 3$. Prove that

$$(3-2a)(3-2b)(3-2c) \leq a^2b^2c^2.$$

Robert Szasz

Day 5 - *24 May 2005*

15 Let $n > 3$ be a positive integer. Consider n sets, each having two elements, such that the intersection of any two of them is a set with one element. Prove that the intersection of all sets is non-empty.

Sever Moldoveanu

16 Let AB and BC be two consecutive sides of a regular polygon with 9 vertices inscribed in a circle of center O. Let M be the midpoint of AB and N be the midpoint of the radius perpendicular to BC. Find the measure of the angle $\angle OMN$.

17 A piece of cardboard has the shape of a pentagon $ABCDE$ in which $BCDE$ is a square and ABE is an isosceles triangle with a right angle at A. Prove that the pentagon can be divided in two different ways in three parts that can be rearranged in order to recompose a right isosceles triangle.

18 Consider two distinct positive integers a and b having integer arithmetic, geometric and harmonic means. Find the minimal value of $|a - b|$.

Mircea Fianu

