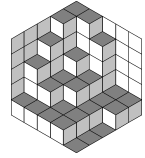




**Romania**  
**Junior Balkan Team Selection Tests**  
Bistrita and Bucharest 2005



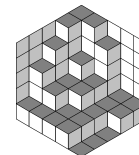
Day 1 - 31 March 2005

- 1] Let  $\mathcal{C}_1(O_1)$  and  $\mathcal{C}_2(O_2)$  be two circles which intersect in the points  $A$  and  $B$ . The tangent in  $A$  at  $\mathcal{C}_2$  intersects the circle  $\mathcal{C}_1$  in  $C$ , and the tangent in  $A$  at  $\mathcal{C}_1$  intersects  $\mathcal{C}_2$  in  $D$ . A ray starting from  $A$  and lying inside the  $\angle CAD$  intersects the circles  $\mathcal{C}_1, \mathcal{C}_2$  in the points  $M$  and  $N$  respectively, and the circumcircle of  $\triangle ACD$  in  $P$ .
- Prove that  $AM = NP$ .
- 2] Find the largest positive integer  $n > 10$  such that the residue of  $n$  when divided by each perfect square between 2 and  $\frac{n}{2}$  is an odd number.
- 3] In a country 6 cities are connected two by two with round-trip air routes operated by exactly one of the two air companies in that country.
- Prove that there exist 4 cities  $A, B, C$  and  $D$  such that each of the routes  $A \leftrightarrow B, B \leftrightarrow C, C \leftrightarrow D$  and  $D \leftrightarrow A$  are operated by the same company.

*Dan Schwartz*



**Romania**  
**Junior Balkan Team Selection Tests**  
Bistrita and Bucharest 2005



**Day 2 - 01 April 2005**

- 4] Let  $a, b, c$  be positive numbers such that  $a + b + c \geq \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ . Prove that

$$a + b + c \geq \frac{3}{abc}.$$

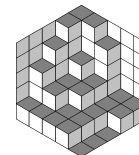
- 5] On the sides  $AD$  and  $BC$  of a rhombus  $ABCD$  we consider the points  $M$  and  $N$  respectively. The line  $MC$  intersects the segment  $BD$  in the point  $T$ , and the line  $MN$  intersects the segment  $BD$  in the point  $U$ . We denote by  $Q$  the intersection between the line  $CU$  and the side  $AB$  and with  $P$  the intersection point between the line  $QT$  and the side  $CD$ .

Prove that the triangles  $QCP$  and  $MCN$  have the same area.

- 6] Let  $ABC$  be an equilateral triangle and  $M$  be a point inside the triangle. We denote by  $A', B', C'$  the projections of the point  $M$  on the sides  $BC, CA$  and  $AB$  respectively. Prove that the lines  $AA', BB'$  and  $CC'$  are concurrent if and only if  $M$  belongs to an altitude of the triangle.



**Romania**  
**Junior Balkan Team Selection Tests**  
Bistrita and Bucharest 2005



**Day 3 - 19 April 2005**

- 7 A phone company starts a new type of service. A new customer can choose  $k$  phone numbers in this network which are call-free, whether that number is calling or is being called. A group of  $n$  students want to use the service.
- (a) If  $n \geq 2k + 2$ , show that there exist 2 students who will be charged when speaking.
- (b) If  $n = 2k + 1$ , show that there is a way to arrange the free calls so that everybody can speak free to anybody else in the group.

*Valentin Vornicu*

- 8 Let  $a, b, c$  be three positive reals such that  $(a + b)(b + c)(c + a) = 1$ . Prove that the following inequality holds:

$$ab + bc + ca \leq \frac{3}{4}.$$

*Cezar Lupu*

- 9 Let  $ABC$  be a triangle with  $BC > CA > AB$  and let  $G$  be the centroid of the triangle. Prove that

$$\angle GCA + \angle GBC < \angle BAC < \angle GAC + \angle GBA.$$

*Dinu Serbanescu*

- 10 Let  $k, r \in \mathbb{N}$  and let  $x \in (0, 1)$  be a rational number given in decimal representation

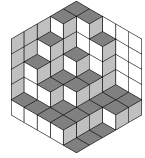
$$x = 0.a_1a_2a_3a_4 \dots$$

Show that if the decimals  $a_k, a_{k+r}, a_{k+2r}, \dots$  are canceled, the new number obtained is still rational.

*Dan Schwarz*



**Romania**  
**Junior Balkan Team Selection Tests**  
Bistrita and Bucharest 2005



**Day 4 - 23 May 2005**

- 11 Three circles  $C_1(O_1)$ ,  $C_2(O_2)$  and  $C_3(O_3)$  share a common point and meet again pairwise at the points  $A$ ,  $B$  and  $C$ . Show that if the points  $A$ ,  $B$ ,  $C$  are collinear then the points  $Q$ ,  $O_1$ ,  $O_2$  and  $O_3$  lie on the same circle.

- 12 Find all positive integers  $n$  and  $p$  if  $p$  is prime and

$$n^8 - p^5 = n^2 + p^2.$$

*Adrian Stoica*

- 13 The positive integers from 1 to  $n^2$  are placed arbitrarily on the  $n^2$  squares of a  $n \times n$  chessboard. Two squares are called *adjacent* if they have a common side. Show that two opposite corner squares can be joined by a path of  $2n - 1$  adjacent squares so that the sum of the numbers placed on them is at least  $\left\lfloor \frac{n^3}{2} \right\rfloor + n^2 - n + 1$ .

*Radu Gologan*

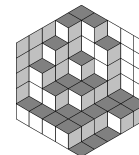
- 14 Let  $a, b, c$  be three positive real numbers with  $a + b + c = 3$ . Prove that

$$(3 - 2a)(3 - 2b)(3 - 2c) \leq a^2b^2c^2.$$

*Robert Szasz*



**Romania**  
**Junior Balkan Team Selection Tests**  
Bistrita and Bucharest 2005



**Day 5 - 24 May 2005**

- [15] Let  $n > 3$  be a positive integer. Consider  $n$  sets, each having two elements, such that the intersection of any two of them is a set with one element. Prove that the intersection of all sets is non-empty.

*Sever Moldoveanu*

- [16] Let  $AB$  and  $BC$  be two consecutive sides of a regular polygon with 9 vertices inscribed in a circle of center  $O$ . Let  $M$  be the midpoint of  $AB$  and  $N$  be the midpoint of the radius perpendicular to  $BC$ . Find the measure of the angle  $\angle OMN$ .

- [17] A piece of cardboard has the shape of a pentagon  $ABCDE$  in which  $BCDE$  is a square and  $ABE$  is an isosceles triangle with a right angle at  $A$ . Prove that the pentagon can be divided in two different ways in three parts that can be rearranged in order to recompose a right isosceles triangle.

- [18] Consider two distinct positive integers  $a$  and  $b$  having integer arithmetic, geometric and harmonic means. Find the minimal value of  $|a - b|$ .

*Mircea Fianu*