THE 22$^{\text{nd}}$
BALKAN MATHEMATICAL OLYMPIAD
Iaşi – May 6$^{\text{th}}$, 2005

Language: *English*

Problem 1. Let ABC be an acute-angled triangle whose inscribed circle touches AB and AC at D and E respectively. Let X and Y be the points of intersection of the bisectors of angles $\angle ACB$ and $\angle ABC$ with the line DE and let Z be the midpoint of BC. Prove that the triangle XYZ is equilateral if and only if $\angle BAC = 60°$.

Problem 2. Find all primes p such that $p^2 - p + 1$ is a perfect cube.

Problem 3. Let a, b, c be positive real numbers. Prove the inequality

$$\frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{a} \geq a + b + c + \frac{4(a-b)^2}{a+b+c}.$$

When does equality occur?

Problema 4. Let $n \geq 2$ be an integer. Let S be a subset of $\{1, 2, \ldots, n\}$ such that S neither contains two elements one of which divides the other, nor contains two elements which are coprime. What is the maximal possible number of elements of such a set S?

Timp de lucru: $4\frac{1}{2}$ ore