**Colegial National "Stefan cel Mare"**
**Targu Neamt**

**OLIMPIADA LOCALA DE MATEMATICA**
**18 ianuarie**
**clasa a XII-a (profil real, matematica- informatica)**

**1)** Fie $(G,\cdot)$ grup multiplicativ si $f, g : G \to G,\ f(x) = x^{2013},\quad g(x) = x^{2014}$ morfisme de grup. Stiind ca *f* este surjectiva sa se arate ca $(G,\cdot)$ este grup abelian.

**2)** Se considera multimea $H = \left\{ \begin{pmatrix} m & n \\ \hat{0} & \hat{1} \end{pmatrix} \middle| m, n \in Z_5, m = \pm\hat{1} \right\}$.

a) Aratati ca H este grup in raport cu inmultirea matricelor patratice de ordinul doi;
b) Determinati numarul elementelor de ordinal doi din grupul $(H,\cdot)$.

**3)** Se considera functiile $f_n : (-3, \infty) \to R,\ f_n(x) = \dfrac{x^n}{x+3}, n \in N$.

a) Aratati ca orice primitiva a functiei $f_4$ este crescatoare;
b) Determinati *n* pentru care functia $f_n$ admite primitive descrescatoare pe intervalul $(-3,0)$.

**4)** Calculati $\int \frac{dx}{\sin x \sin(x+1) \sin(x+2) \sin(x+3)}$, *x* fiind dintr-un interval in care numitorul nu se anuleaza.

**Nota:**
**Toate subiectele sunt obligatorii.**
**Timp de lucru: 3 ore**
**Fiecare subiect rezolvat corect se noteaza cu 7 puncte.**

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**Barem de corectare si notare**

**1)** $f(xy)=(xy)^{2013} = f(x)f(y)=x^{2013}y^{2013}, \forall x,y \in G$ ........................................................1p

$g(xy)=(xy)^{2014} = g(x)g(y)=x^{2014}y^{2014}, \forall x,y \in G$ ........................................................1p

$x^{2014}y^{2014} = (xy)^{2014} = (xy)^{2013}(xy)=x^{2013}y^{2013}xy$ de unde simplificand obtinem ………2p

$xy^{2013} = y^{2013}x, \forall x,y \in G$ ........................................................................................ 1p

Cum functia *f* este surjectiva $\exists a \in G$, *pentru care* $y^{2013} = f(a)$...................................1p

$xf(a)= f(a)x, \forall x, f(a)\in G$ …………………………………………………………..1p

**2)** a) Fie $X,Y \in H, X = \begin{pmatrix} x_1 & x_2 \\ \hat{0} & \hat{1} \end{pmatrix}, Y = \begin{pmatrix} y_1 & y_2 \\ \hat{0} & \hat{1} \end{pmatrix}$ ……………………………………………1p

Atunci $XY = \begin{pmatrix} x_1y_1 & x_1y_2 + x_2 \\ \hat{0} & \hat{1} \end{pmatrix}$ ……………………………………………………1p

Cum $x_1, y_1 \in \{-\hat{1},\hat{1}\}$ rezulta $x_1y_1 \in \{-\hat{1},\hat{1}\}$. Deci $X,Y \in H$ ..........................................1p

Pentru *H* grup in raport cu inmultirea matricelor.........................................................1p

b) $X = \begin{pmatrix} m & n \\ \hat{0} & \hat{1} \end{pmatrix}$, $ord(X)=2 \Leftrightarrow X^2 = I_2, X \neq I_2$ ..........................................................1p

$\begin{pmatrix} m^2 & mn \\ \hat{0} & \hat{1} \end{pmatrix} = I_2$, de unde $(m,n)\in \{(\hat{1},\hat{0}),(\hat{4},n)\}, n \in Z_5$ ..................................................1p

Deci H are *5* elemente de ordin *2*.......................................................................1p

**3)** a) Fie $F_4$ o primitva a lui a lui $f_4$. $F_4$ derivabila si $F_4'(x)=\dfrac{x^4}{x+3} \geq 0, \forall x \in (-3,\infty)$ ....2p

$F_4$ este crescatoare…………………………………………………………….........…...1p

b) Fie $F_n$ o primitva a lui a lui $f_n$. $F_n$ derivabila.........................................................2p

$F_n'(x)=x^{n-1}\dfrac{x}{x+3} < 0, \forall x \in (-3,0) \Rightarrow$

$x^{n-1} > 0, \forall x \in (-3,0) \Rightarrow$ ............................................................2p

$n$ *impar*

.............................................................................................................................1p

**4)** $ctg\alpha - ctg\beta = \frac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$ de unde $\frac{1}{\sin\alpha\sin\beta} = \frac{1}{\sin(\alpha-\beta)}(ctg\alpha - ctg\beta), \alpha \neq \beta$ ....**1p**

$I = \int \frac{dx}{\sin x\sin(x+1)\sin(x+2)\sin(x+3)} = \frac{1}{\sin^2 1}\int [ctgx - ctg(x+1)][ctg(x+2) - ctg(x+3)]dx$ ..........**1p**

$= \frac{1}{\sin^2 1}\int [ctgxctg(x+2) - ctg(x+1)ctg(x+2) - ctgxctg(x+3) + ctg(x+1)ctg(x+3)]dx$ ..**1p**

Din $ctg(\alpha-\beta) = \frac{ctg\alpha ctg\beta + 1}{ctg\beta - ctg\alpha}$ avem $ctg\alpha ctg\beta = ctg(\alpha-\beta)(ctg\beta - ctg\alpha) - 1$**..............1p**

$I = \frac{1}{\sin^2 1}\int \{ctg2[ctg(x+2) - ctgx] - 1 + ctg1[ctg(x+2) - ctg(x+1)] + 1 + ctg3[ctg(x+3) - ctgx] + 1 - ctg2[ctg(x+3) - ctg(x+1)] - 1\}dx$

..........................................**1p**

$I = \frac{1}{\sin^2 1}[(ctg2 - ctg3)\ln|\sin x| + (ctg2 - ctg1)\ln|\sin(x+1)| + (ctg1 - ctg2)\ln|\sin(x+2)| + (ctg3 - ctg2)\ln|\sin(x+3)|] + c$

..........................................**2p**