



**Olimpiada de Astronomie și Astrofizică**  
**Etapa Națională 2013**  
**Proba de Analiză a Datelor**  
**Barem de evaluare**  
**Seniori**



Barem	Parțial	Punctaj
<b>Subiectul I</b>		<b>10</b>
<p>a. Elipsa</p> $v_1 = \frac{m_1 + m_2}{m_1} \sqrt{2K \frac{M}{R} \left( \frac{3m_2^2}{2(m_1 + m_2)^2} - 1 \right)};$ $v_{\max} = \frac{2m_2}{m_1 + m_2} \sqrt{K \frac{M}{R}}; r_{\min} = \frac{R}{2}; v_{\min} = \frac{r_{\min} v_{\max}}{r_{\max}};$ $r_{\max} = R \frac{\sqrt{(m_1 + m_2)^4 + 4m_2^2(m_2^2 - (m_1 + m_2)^2)} - (m_1 + m_2)^2}{4(m_2^2 - (m_1 + m_2)^2)}.$	1 p  1 p  2 p	
<p>b.</p> $v_{1,\min} = \frac{1}{m_1} \sqrt{\frac{KM}{R} (2(m_1 + m_2)^2 - m_2^2)};$ $-K \frac{M}{r_{\min}} + \frac{1}{2} v_{\max}^2 = 0; r_{\min} = \frac{m_2^2 R}{2(m_1 + m_2)^2};$ $v_{\max} = \frac{2(m_1 + m_2)}{m_2} \sqrt{\frac{KM}{R}}.$	1,5  1,0  0,5	
<p>c.</p> $\cos \frac{\theta}{2} = \frac{m_2}{\sqrt{2}(m_1 + m_2)}.$ $t_0 = 0; t = \frac{p^2}{2C} \left( u + \frac{u^3}{3} \right); \tan \frac{\theta}{2} = u;$ $p = \frac{R^2 v^2 \sin^2 \alpha}{KM} = \frac{R m_2^2}{(m_1 + m_2)^2},$	1  2	
<b>Subiectul II</b>		<b>10</b>
<p>Tabel date observationale</p> $\sin p_{\text{an}} = \frac{a_0}{\Delta} \cong p_{\text{an}}; \Delta = \frac{a_0}{p_{\text{an}}},$ $M = m + 5 - 5 \log  \Delta ,$ $R = \Delta \cos g.$	2  2  2  4	



**Olimpiada de Astronomie și Astrofizică**  
**Etapa Națională 2013**  
**Proba de Analiză a Datelor**  
**Barem de evaluare**  
**Seniori**



Barem	Parțial	Punctaj
<b>Subiectul III</b>		<b>10</b>
$T^2 = 4\pi^2 \frac{r^{p+1}}{KM},$ $\log T  = \frac{p+1}{2} \log r  + \frac{1}{2} \log \frac{4\pi^2}{KM}.$ <p>Din graficul dependentei, <math>\log T  = f(\log r )</math>.</p> $\tan \alpha = \frac{\Delta(\log T )}{\Delta(\log r )} = \frac{p+1}{2} = \frac{1,5}{1} = 1,5;$ $p = 2.$	4  2  2  2	
<b>Subiectul IV</b>		<b>10</b>
<p><b>Rezolvare</b> : 10 puncte:</p> <p>a) (3 puncte)  reprezentarea triunghiului sferic (0.5 puncte)  Pentru determinarea formulei se va folosi triunghiul sferic corespunzător:  Din teorema sinusurilor:</p> <div style="text-align: center;"> </div> $\frac{\sin \theta}{\sin(90^\circ - \delta)} = \frac{\sin(\alpha - \alpha_1)}{\sin x}$ $\sin x = \frac{\cos \delta \sin(\alpha - \alpha_1)}{\sin \theta} \quad (1) \quad (0.5 \text{ puncte})$ <p>Din teorema cosinusurilor:</p> $\cos x = \sin \delta \sin \delta_1 + \cos \delta \cos \delta_1 \cos(\alpha - \alpha_1) \quad (2)$ $\sin \delta = \sin \delta_1 \cos x + \cos \delta_1 \sin x \cos \theta \quad (3)$ <p>Din (1), (2) și (3):</p>		



**Olimpiada de Astronomie și Astrofizică**  
**Etapa Națională 2013**  
**Proba de Analiză a Datelor**  
**Barem de evaluare**  
**Seniori**



<b>Barem</b>	<b>Parțial</b>	<b>Punctaj</b>
<p> <math display="block">\sin \delta = \sin^2 \delta_1 \sin \delta + \cos \delta \cos \delta_1 \sin \delta_1 \cos(\alpha - \alpha_1) + \cos \delta \cos \delta_1 \sin(\alpha - \alpha_1) \operatorname{ctg} \theta</math>           (0.5puncte)         </p> <p>Impartind prin <math>\cos \delta</math> obținem:</p> <p> <math display="block">\operatorname{tg} \delta = \sin^2 \delta_1 \tan \delta + \cos \delta_1 \sin \delta_1 \sin(\alpha - \alpha_1) + \cos \delta_1 \sin(\alpha - \alpha_1) \operatorname{ctg} \theta</math>           (0,25 puncte)         </p> <p> <math display="block">\operatorname{tg} \delta = \frac{\sin \delta_1 \sin(\alpha - \alpha_1) + \sin(\alpha - \alpha_1) \operatorname{ctg} \theta}{\cos \delta_1}</math> </p> <div style="text-align: center; margin: 10px 0;"> </div> <p>Valoare numerica: <math>\delta = 4.40^0</math> (0.25 puncte)</p> <p>b)(2,5 puncte)          reprezentarea triunghiului sferic (0.5 puncte)          In triunghiul sferic figurat se aplica succesiv teorema cosinusurilor:  <math>\cos \lambda = \sin \delta_1 \sin D + \cos \delta_1 \cos D \cos(A - \alpha_1)</math> in triunghiul sferic dat si se gasesc valorile numerice.(figurate in tabelul final); 2 puncte=0,25puncte*8;</p> <p>c) (1 punct) Se aplica formula <math>V = \frac{\sum_i V_r \cos \lambda_i}{\sum_i \cos^2 \lambda_i}</math> si se gaseste valoarea numerica.(in tabelul final);</p> <p>d) (2 puncte)          Se aplica succesiv formula:  <math>V_t = V \sin \lambda</math> (valorile numerice figurate in tabelul final) (1punct)          Iar distanta dintre Pamant si fiecare nava se gaseste in felul urmator  <math>d = \frac{V_t (km/s)}{4.74 \mu (" / an)}</math> (pc) (valorile numerice figurate in tabelul final) (1 punct);          e) (0,5 puncte) Dupa ce am determinat</p> <div style="text-align: right; margin-top: 20px;"> </div>		



**Olimpiada de Astronomie și Astrofizică**  
**Etapa Națională 2013**  
**Proba de Analiză a Datelor**  
**Barem de evaluare**  
**Seniori**



<b>Barem</b>	<b>Par- țial</b>	<b>Punc- taj</b>																																																																																
<p>aceste distante facem media aritmetica a lor (rezultat figurat in tabelul final) (0.5 puncte);</p> <p>f. (1 punct) Timpul de la plecare este <math>T = \frac{d}{V}</math></p> <p>Valoare numerica: 1087733.5839 ani</p> <p>Ipoteza este gresita pentru ca acum 1 milion de ani nu existau guverne secrete.</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>Ascensie dreapta</th> <th>Declinație</th> <th><math>\mu</math></th> <th><math>\theta</math></th> <th><math>V_r</math></th> <th><math>\lambda</math></th> <th><math>V_t</math></th> <th><math>d</math></th> </tr> <tr> <th><i>ore</i></th> <th><i>grade</i></th> <th><i>ms de arc /an</i></th> <th><i>rad</i></th> <th><i>km/s</i></th> <th></th> <th></th> <th></th> </tr> </thead> <tbody> <tr> <td>3.886125</td> <td>17.327083</td> <td>147.05</td> <td>1.776</td> <td>35</td> <td><b>41.41798</b></td> <td><b>31.716</b></td> <td><b>45.50248</b></td> </tr> <tr> <td>3.918472</td> <td>16.998472</td> <td>150.48</td> <td>1.738</td> <td>36.8</td> <td><b>40.90528</b></td> <td><b>31.39303</b></td> <td><b>44.0125</b></td> </tr> <tr> <td>3.994558</td> <td>10.330417</td> <td>133.11</td> <td>1.597</td> <td>39.1</td> <td><b>39.10323</b></td> <td><b>30.23806</b></td> <td><b>47.9253</b></td> </tr> <tr> <td>4.188967</td> <td>5.523056</td> <td>146.95</td> <td>1.537</td> <td>36.6</td> <td><b>36.26984</b></td> <td><b>28.36205</b></td> <td><b>40.7183</b></td> </tr> <tr> <td>4.241783</td> <td>22.451861</td> <td>101.02</td> <td>1.915</td> <td>38.4</td> <td><b>37.83302</b></td> <td><b>29.40591</b></td> <td><b>61.41139</b></td> </tr> <tr> <td>4.240458</td> <td>14.625028</td> <td>107.46</td> <td>1.757</td> <td>39.2</td> <td><b>35.93264</b></td> <td><b>28.13407</b></td> <td><b>55.23412</b></td> </tr> <tr> <td>4.275967</td> <td>21.907472</td> <td>112.2</td> <td>1.915</td> <td>38.5</td> <td><b>37.20648</b></td> <td><b>28.99011</b></td> <td><b>54.5103</b></td> </tr> <tr> <td>4.306444</td> <td>21.579306</td> <td>102.21</td> <td>1.909</td> <td>37.8</td> <td><b>36.69849</b></td> <td><b>28.65042</b></td> <td><b>59.137</b></td> </tr> </tbody> </table> <p><math>V = 47,94215</math></p> <p>AVG (D) = 51,05642</p>	Ascensie dreapta	Declinație	$\mu$	$\theta$	$V_r$	$\lambda$	$V_t$	$d$	<i>ore</i>	<i>grade</i>	<i>ms de arc /an</i>	<i>rad</i>	<i>km/s</i>				3.886125	17.327083	147.05	1.776	35	<b>41.41798</b>	<b>31.716</b>	<b>45.50248</b>	3.918472	16.998472	150.48	1.738	36.8	<b>40.90528</b>	<b>31.39303</b>	<b>44.0125</b>	3.994558	10.330417	133.11	1.597	39.1	<b>39.10323</b>	<b>30.23806</b>	<b>47.9253</b>	4.188967	5.523056	146.95	1.537	36.6	<b>36.26984</b>	<b>28.36205</b>	<b>40.7183</b>	4.241783	22.451861	101.02	1.915	38.4	<b>37.83302</b>	<b>29.40591</b>	<b>61.41139</b>	4.240458	14.625028	107.46	1.757	39.2	<b>35.93264</b>	<b>28.13407</b>	<b>55.23412</b>	4.275967	21.907472	112.2	1.915	38.5	<b>37.20648</b>	<b>28.99011</b>	<b>54.5103</b>	4.306444	21.579306	102.21	1.909	37.8	<b>36.69849</b>	<b>28.65042</b>	<b>59.137</b>		
Ascensie dreapta	Declinație	$\mu$	$\theta$	$V_r$	$\lambda$	$V_t$	$d$																																																																											
<i>ore</i>	<i>grade</i>	<i>ms de arc /an</i>	<i>rad</i>	<i>km/s</i>																																																																														
3.886125	17.327083	147.05	1.776	35	<b>41.41798</b>	<b>31.716</b>	<b>45.50248</b>																																																																											
3.918472	16.998472	150.48	1.738	36.8	<b>40.90528</b>	<b>31.39303</b>	<b>44.0125</b>																																																																											
3.994558	10.330417	133.11	1.597	39.1	<b>39.10323</b>	<b>30.23806</b>	<b>47.9253</b>																																																																											
4.188967	5.523056	146.95	1.537	36.6	<b>36.26984</b>	<b>28.36205</b>	<b>40.7183</b>																																																																											
4.241783	22.451861	101.02	1.915	38.4	<b>37.83302</b>	<b>29.40591</b>	<b>61.41139</b>																																																																											
4.240458	14.625028	107.46	1.757	39.2	<b>35.93264</b>	<b>28.13407</b>	<b>55.23412</b>																																																																											
4.275967	21.907472	112.2	1.915	38.5	<b>37.20648</b>	<b>28.99011</b>	<b>54.5103</b>																																																																											
4.306444	21.579306	102.21	1.909	37.8	<b>36.69849</b>	<b>28.65042</b>	<b>59.137</b>																																																																											