COLEGIUL NATIONAL "STEFAN CEL MARE"

TG. NEAMT

OLIMPIADA DE MATEMATICA – ETAPA LOCALA

CLASA a X-a – matematica-informatica

1. Fie x, y, z trei numere reale strict pozitive , a>1. Sa se demonstreze inegalitatea:

$$\boldsymbol{a^{\frac{xy}{z}} + a^{\frac{yz}{x}} + a^{\frac{zx}{y}} \geq a^x + a^y + a^z}$$

Prof. Iosub Maria

2. A. Arătați ca pentru orice $\boldsymbol{z_1}, \boldsymbol{z_2} \in \mathrm{C}$ avem

$$\frac{|\boldsymbol{z_2}|\boldsymbol{z_1} + |\boldsymbol{z_1}|\boldsymbol{z_2}}{|\boldsymbol{z_1 z_2}| + \boldsymbol{z_1 z_2}} \in \boldsymbol{R}$$

B. Arătați ca pentru orice $\boldsymbol{z_1}, \boldsymbol{z_2}, \boldsymbol{z_3} \in \mathrm{C}$, cu $|\boldsymbol{z_1}| = |\boldsymbol{z_2}| = |\boldsymbol{z_3}|=1$ avem

$$\left|\frac{z_1+z_2+z_3+z_1z_2z_3}{1+z_1z_2+z_2z_3+z_3z_1}\right|=1$$

3. Rezolvați in C ecuația:

$$\left(1 + \frac{iz}{n}\right)^n + \left(1 - \frac{iz}{n}\right)^n = 0 \text{ unde n} \in N^*$$

4. Daca numerele x, y, z ∈ $(1, \infty)$, sa se arate ca:

$$\left(\log_x \frac{y+z}{2}\right)\left(\log_y \frac{z+x}{2}\right)\left(\log_z \frac{x+y}{2}\right) \geq 1$$

Nota: Toate subiectele sunt obligatorii.

Fiecare subiect este punctat cu 7 puncte.

Timp de lucru – 3 ore.

Colegiul Naţional „Ştefan cel Mare”

Tg. Neamţ

Olimpiada locală de matematică

18 ianuarie 2014

Clasa a X-a

BAREM – mate info

1. $\boldsymbol{a}^{\frac{xy}{z}} + \boldsymbol{a}^{\frac{yz}{x}} + \boldsymbol{a}^{\frac{zx}{y}} \geq \boldsymbol{a}^{x} + \boldsymbol{a}^{y} + \boldsymbol{a}^{z}$

Din $\boldsymbol{a}^{\frac{xy}{z}} + \boldsymbol{a}^{\frac{yz}{x}} \geq 2\sqrt{\boldsymbol{a}^{\frac{xy}{z}} \cdot \boldsymbol{a}^{\frac{yz}{x}}} = 2\sqrt{a^{\frac{xy}{z}+\frac{yz}{x}}} = 2\sqrt{a^{2\sqrt{\frac{xy}{z}\cdot\frac{yz}{x}}}} = 2a^{y}$ si analoagele rezulta inegalitatea de demonstrat. 7p

2. A. Fie $z = \frac{|z_2|z_1 + |z_1|z_2}{|z_1z_2| + z_1z_2}$. Observam ca $\bar{z} = \frac{|z_2|\overline{z_1} + |z_1|\overline{z_2}}{|z_1z_2| + \overline{z_1z_2}} = \frac{|z_2|\frac{|z_1|^2}{z_1} + |z_1|\frac{|z_2|^2}{z_2}}{|z_1z_2| + \frac{|z_1|^2|z_2|^2}{z_1\ \ z_2}} =$

$= \frac{|z_1z_2||z_1|z_2 + |z_1z_2||z_2|z_1}{|z_1z_2|z_1z_2 + |z_1z_2|} = \frac{|z_1|z_2 + |z_2|z_1}{z_1z_2 + |z_1z_2|} = z \Rightarrow z \in R$ 4p

B. Fie $z_0 = \frac{z_1+z_2+z_3+z_1z_2z_3}{1+z_1z_2+z_2z_3+z_3z_1}$. Deoarece

$$\overline{\mathbf{z_0}} = \frac{\overline{z_1}+\overline{z_2}+\overline{z_3}+\overline{z_1z_2z_3}}{1+\overline{z_1z_2}+\overline{z_2z_3}+\overline{z_3z_1}} = \frac{\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}+\frac{1}{z_1z_2z_3}}{1+\frac{1}{z_1z_2}+\frac{1}{z_2z_3}+\frac{1}{z_3z_1}} = \frac{1+z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3+z_1z_2z_3} = \frac{1}{z_0} \Rightarrow z_0\overline{\mathbf{z_0}} = 1$$

$\Rightarrow |z_0| = 1$ 3p

3. Z=-ni nu este solutie.Atunci ecuatia este echivalenta cu

$\left(\frac{n+iz}{n-iz}\right)^n =$

$\cos\pi + i\sin\pi$, *de unde se obtine solutia* $Z_k = n\tan\frac{(2k+1)n}{2n}, k \in$

$\{0,1,2,\ldots,n-1\}$ 3p

4. Aplicarea inegalitatii mediilor. 3p

   Aplicarea proprietatii logaritmilor. 3p

   Finalizare. 1p