Barem clasa a VII-a
(OLM 2016-etapa locală)

Subiectul I. (7 puncte)

a) $\overline{14a_1b_1c_1} \,\vdots\, 196$, $\overline{14a_2b_2c_2} \,\vdots\, 196$, $\overline{14a_3b_3c_3} \,\vdots\, 196$ **(1 punct)**

$14112 = 196 \cdot 72$, $14308 = 196 \cdot 73$, $14504 = 196 \cdot 74$ **(1 punct)**

$\sqrt{14112 + 14308 + 14504} = \sqrt{196 \cdot 219} = 14\sqrt{219}$ **(1 punct)**

$\overline{a_1b_1c_1} = 112$, $\overline{a_2b_2c_2} = 308$, $\overline{a_3b_3c_3} = 504$ **(1 punct)**

b)

$$\sqrt{\frac{1}{9}\left(10 + \frac{11}{2} + \frac{12}{3} + \cdots + \frac{73}{64}\right) - \frac{1}{9}\left(9 + \frac{9}{2} + \cdots + \frac{9}{64}\right)} =$$

$$\sqrt{\frac{1}{9}\left(10 + \frac{11}{2} + \frac{12}{3} + \cdots + \frac{73}{64} - 9 - \frac{9}{2} - \cdots - \frac{9}{64}\right)} = \sqrt{\frac{1}{9}(\underbrace{1 + 1 + \cdots + 1}_{64})} = \sqrt{\frac{64}{9}} = \frac{8}{3}$$

(3 puncte)

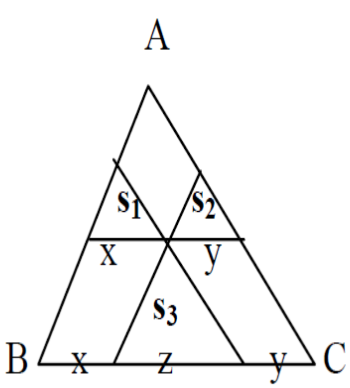
Subiectul II. (7 puncte)

a) Calcul direct **(3 puncte)**

b) Folosim $1 - \frac{1}{n^2} = \frac{n-1}{n}\frac{n+1}{n}$ şi $1 - \frac{1}{n^k} \geq 1 - \frac{1}{n^2}, \forall n \geq 2$ **(2 puncte)**

$$\left(1 - \frac{1}{2^k}\right)\left(1 - \frac{1}{3^k}\right)\ldots\left(1 - \frac{1}{2016^k}\right) \geq \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\ldots\left(1 - \frac{1}{2016^2}\right) = \frac{2017}{2016 \cdot 2} = \frac{2017}{4032} > \frac{2017}{4034} = \frac{1}{2}$$ **(2 puncte)**

Subiectul III. (7 puncte)



$$\frac{s_1}{S} = \left(\frac{x}{x+y+z}\right)^2 \Rightarrow \frac{\sqrt{s_1}}{\sqrt{S}} = \frac{x}{x+y+z}$$ **(2 puncte)**

$$\frac{s_2}{S} = \left(\frac{y}{x+y+z}\right)^2 \Rightarrow \frac{\sqrt{s_2}}{\sqrt{S}} = \frac{y}{x+y+z}$$ **(2 puncte)**

$$\frac{s_3}{S} = \left(\frac{z}{x+y+z}\right)^2 \Rightarrow \frac{\sqrt{s_3}}{\sqrt{S}} = \frac{z}{x+y+z}$$ **(2 puncte)**

$$\Rightarrow \frac{\sqrt{s_1} + \sqrt{s_2} + \sqrt{s_3}}{\sqrt{S}} = 1$$

Prin urmare $S = \left(\sqrt{s_1} + \sqrt{s_2} + \sqrt{s_3}\right)^2$ **(1 punct)**

Subiectul IV. (7 puncte)

a) Din ipoteză $\Rightarrow$ m($\sphericalangle EFP$) = 75^0, m($\sphericalangle$ EPF) = 15^o, ducând mediana în triunghiul EFP , EM este jumătate din mediană, care este jumătate din ipotenuză $\Rightarrow$ EM = $\frac{1}{4} \cdot FP \Rightarrow \frac{EM}{FP} = \frac{1}{4}$. (1). **(4 puncte)**

b) Fie $BN \perp AD$, $N \in AD$. Avem $\Delta DNB \sim \Delta DME$ (cazul UU) $\Rightarrow \frac{EM}{BN} = \frac{ED}{BD}$. **(1 punct)**

Din ipoteză deducem că $\frac{ED}{BD} = \frac{1}{4}$. Ultimele două egalități și 1), ne dau $\frac{EM}{FP} = \frac{EM}{BN} \Rightarrow$ FP = BN **(1 punct)**

$\Rightarrow A_{romb} = AD \cdot BN = AB \cdot FP$ **(1 punct)**