Barem clasa a VIII-a
(OLM 2016-etapa locală)

Subiectul I. (7 puncte)

a) Se arată prin calcul direct; **(2 puncte)**

b) Din a) avem relația $(a+b+c)\cdot(a^2+b^2+c^2-ab-bc-ac)=0$ **(1 punct)**

Cum a,b,c sunt cifre ale unui număr natural de trei cifre $\Rightarrow a+b+c\neq 0$ **(1 punct)**

Atunci $a^2+b^2+c^2-ab-bc-ac=0 \Leftrightarrow (a-b)^2+(b-c)^2+(c-a)^2=0 \Leftrightarrow a=b=c$ **(2 puncte)**

Numerele de trei cifre, cu cifre egale sunt 111,222,333,444,…,999.

Dar $111=3\cdot 37$, deci toate numerele de trei cifre, cu cifre egale se divid cu 37 **(1 punct)**

Subiectul II. (7 puncte)

a) Observăm că fiecare termen al sumei este de forma $\dfrac{k^4+3k^2+1}{k^3+k}$ şi avem: **(1 punct)**

$\dfrac{k^4+3k^2+1}{k^3+k}=\dfrac{k^4+2k^2+1}{k^3+k}+\dfrac{k^2}{k^3+k}=\dfrac{\left(k^2+1\right)^2}{k\left(k^2+1\right)}+\dfrac{k^2}{k\left(k^2+1\right)}=\dfrac{k^2+1}{k}+\dfrac{k}{k^2+1}>2$,pentru $k\in\left\{\sqrt{1},\sqrt{2},...,\sqrt{1008}\right\}$ **(1 punct)**

Deci $S>2\cdot 1008$, adică $S>2016$. **(1 punct)**

b) $(x-\sqrt{2})^2+(y-\sqrt{3})^2=0 \Rightarrow x=\sqrt{2}, y=\sqrt{3} \Rightarrow I=[\sqrt{2},\sqrt{3}]$ **(2 puncte)**

$\sqrt{2}\leq\dfrac{n}{100}\leq\sqrt{3} \Leftrightarrow n\in[100\sqrt{2},100\sqrt{3}]$, Dar $\boldsymbol{n}\in N \Rightarrow n\in\{142,143,\dots,173\}$, Deci card A∩I = 32 **(2 puncte)**

Subiectul III. (7 puncte)

a) $AA_1=A_1A_2=\ldots=A_{99}C=\dfrac{3}{25}$

$EF\perp(ABC)$, $EM\perp AC$; $FM, AC\subset(ABC)\Rightarrow FM\perp AC$, $m(\angle EMF)=60^\circ$

$MF=\dfrac{5\sqrt{3}}{2}$cm , $AM=\dfrac{5}{2}$ **(2 puncte)**

$n\cdot\dfrac{3}{25}<\dfrac{5}{2}<(n+1)\cdot\dfrac{3}{25} \Leftrightarrow n<20\dfrac{5}{6}<n+1 \Rightarrow n=20 \Rightarrow M\in\left[A_{20}A_{21}\right]$ **(2 puncte)**

b) $PF\perp EF$, $PF\perp AB$, $AB\cap EF=\{F\} \Rightarrow PF\perp(ABE)$

$AP=2AF=10$cm , **(2 puncte)**

$n=83,(3) \Rightarrow M\in\left[A_{83}A_{84}\right]$ **(1 punct)**

Subiectul IV. (7 puncte)

Folosim inegalitatea mediilor $\sqrt{a\cdot 1}\leq\dfrac{a+1}{2}$, $\sqrt{b\cdot 1}\leq\dfrac{b+1}{2}$, $\sqrt{c\cdot 1}\leq\dfrac{c+1}{2}$ **(2 puncte)**

$$A=\sqrt{a}\cdot(b+c)+\sqrt{b}\cdot(a+c)+\sqrt{c}\cdot(a+b)\leq\frac{a+1}{2}\cdot(b+c)+\frac{b+1}{2}\cdot(a+c)+\frac{c+1}{2}\cdot(a+b)=$$

Avem $=ab+ac+bc+a+b+c=\dfrac{(a+b+c)^2-(a^2+b^2+c^2)}{2}+(a+b+c)=$ **(2 puncte)**

$$=\frac{(a+b+c)^2+2\cdot(a+b+c)-d^2}{2}.$$

Din $(a+b+c)^2=a^2+b^2+c^2+2\cdot(ab+ac+bc)$ şi $ab+ac+bc\leq a^2+b^2+c^2$ **(2 puncte)**

Rezultă că $(a+b+c)^2\leq 3\cdot(a^2+b^2+c^2)=3\cdot d^2=9$, $a+b+c\leq 3$. Deci $A\leq\dfrac{3^2+2\cdot 3-3}{2}=6$. **(1 punct)**