BAREME SUBIECTE-O.N.M.-FEBRUARIE 2016
-FAZA LOCALA-

1. a) Rezolvati in **Z** ecuatia : 5·(2·I3x-4I+4)-30=10

Solutie : 5(2I3x-4I+4)=40 → 2I3x-4I+4=8 → I3x-4I=2 → x=2∈ $\mathbb{Z}$ si x=$\frac{2}{3}$ ∉ $\mathbb{Z}$ (2 puncte)

b) Daca x= $\sqrt{2010+(2+4+6+\cdots+4018)}$, sa se arate ca x∈ **N** .

Solutie : x=$\sqrt{2010+2(1+2+\cdots 2009)}=\sqrt{2010+2009\cdot 2010}=\sqrt{2010\cdot 2010}=$ $2010 \in \mathbb{N}$ (3 puncte)

c) Fie a,b,c,d numere reale pozitive astfel incat abcd=1.Calculati :

E=$\frac{7+a}{1+a+ab+abc}+\frac{7+b}{1+b+bc+bcd}+\frac{7+c}{1+c+cd+cda}+\frac{7+d}{1+d+da+dab}$.

Solutie :

$$\frac{7+a}{1+a+ab+abc}^{d)}+\frac{7+b}{1+b+bc+bcd}+^{b)}\frac{7+c}{1+c+cd+cda}+\frac{7+d}{1+d+da+dab}\overset{bc)}{=}\frac{7d+ad+7+d}{1+d+da+dab}+\frac{7b+bc+7+b}{1+b+bc+bcd}=\frac{8+8b+8bc+8bcd}{1+b+bc+bcd}=8 .$$

(2 puncte)

2. Se considera numarul a_n=18$\underbrace{77\ldots77}_{de\ n\ ori}$889 , cu n numar natural , si c_n catul impartirii numarului a_n la 13.

a) Sa se arate ca a_n se divide cu 13 pentru oricare n.

b) Sa se determine n pentru care s(a_n)=2s(c_n) , unde s(m) reprezinta suma cifrelor numarului m.

Solutie :

a) a_n=18· $10^{n+3}+7\cdot 10^3\cdot(10^{n-1}+10^{n-2}+\cdots+10+1)+889=13\cdot 10^{n+3}+13\cdot 68+$ $7\cdot 10^3\cdot\frac{10^n-1}{9}+5\cdot 10^{n+3}+5.$

Dar , $7\cdot 10^3\cdot\frac{10^n-1}{9}+5\cdot 10^{n+3}+5=\frac{52\cdot 10^{n+3}-6955}{9}=\frac{13\cdot(4\cdot 10^{n+3}-535)}{9}=\frac{13}{9}\cdot\underbrace{\overline{399\ldots 9465}}_{n+4\ cifre}=$

$13\cdot\underbrace{\overline{44\ldots 4385}}_{n+3\ cifre}$. Deci a_n=$13\cdot\left(10^{n+3}+68+\underbrace{\overline{44\ldots 4385}}_{n+3\ cifre}\right)$, avem ca a_n⋮ 13.

4 puncte

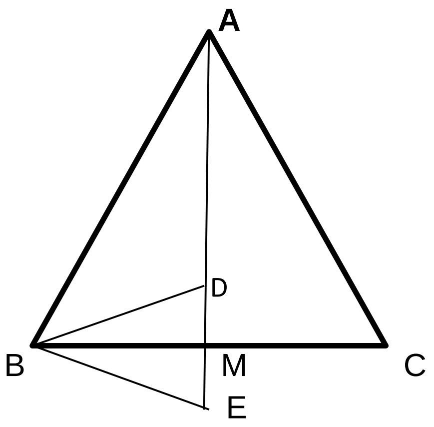
b) s(a_n)=1+8+7n+8+8+9=7n+34

c_n=$\underbrace{\overline{144\ldots 4453}}_{n+4\ cifre}$ ⇛ s(c_n)=9+4· $(n+1)=4n+13$ ⇛ $7n+34=2\cdot(4n+13)$ ⇛ $n=8$

3 puncte

3.Fie ABC un triunghi echilateral , M mijlocul laturii [BC] si D∈(AM) astfel incat AM+MD=AB.Sa se determine masura unghiului $\sphericalangle DBM$.

Solutie :



Fie E∈ $(AM\ astfel\ incat\ AE = AB \ \Rightarrow AM + ME = AB = AM + MD \ \Rightarrow ME = MD$.

$$\Delta ABE\ isoscel\ (AB = AE), m(\sphericalangle\ BAE) = 30^0 \Rightarrow m(\sphericalangle\ ABE) = \frac{180-30}{2} = 75^0 \ \Rightarrow m(\sphericalangle\ MBE) = 75^0 - 60^0 = 15^0.\ \Delta BDM \equiv \Delta BEM (cazul\ C.C.) \Rightarrow m(\sphericalangle\ DBM) = 15^0.$$

7 puncte

4. Fie ABCD paralelogram in care AB>BC , [AE bsectoarea unghiului A , [BF bisectoarea unghiului B (E,F∈ (DC)), X mijlocul segmentului [AE] , iar Y mijlocul segmentului [BF].

a) Demonstrati ca DXYF este paralelogram .

b) Daca 5AD=3AB si XY=24 cm , aflati perimetrul paralelogramului ABCD .

Solutie :

a) Δ ADE si ΔBCF isoscele →DE=AD=BC=CF → DF=EC.
ABEF trapez si X,Y mijloace → XY ∥ AB ∥ DF → XY=$\frac{AB-EF}{2} = \frac{DC-EF}{2} = DF$.
DE=AD=CF=AB → DF=EC → DXYF parallelogram . (4 puncte)

b) Notam EF=x → DF=EC=24 → AB=48+x , AD=24+x
5(24+x)=3(48+x) → x=12 → P_{ABCD}=192 cm. (3 puncte)