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OLIMPIADA NAȚIONALĂ DE MATEMATICĂ

ETAPA LOCALĂ , CLASA a VIII - a

22 FEBRUARIE 2014

SUBIECTUL I

a) **Daca** $a,b,A,B\in\mathbb{R}$ **astfel incat** $a=A+1$ **si** $b=B+1$**, atunci demonstrati ca:** $a^2+b^2-a-b-ab=\dfrac{(A-B)^2+A^2+B^2-2}{2}$

b) **Determinati toate perechile** (x,y) **de numere intregi nenule pentru care** $x^2+y^2=x+y+xy$

Prelucrare GM/2013

SUBIECTUL II

a) **Pentru orice** $y\in\mathbb{R}^*$**, demonstrati ca:**

$$y^2+\frac{1}{y^2}-y-\frac{1}{y}=\frac{(y-1)(y^3-1)}{y^2}$$

b) **Fie** $a\in\mathbb{R}, b\in\mathbb{R}, 0<a<b$ **si expresia** $E(x)=\dfrac{x-a}{b-x}+\dfrac{b-x}{x-a}, x\in\mathbb{R}-\{a,b\}$

Demonstrati ca i) $E(\dfrac{2ab}{a+b})>E(\dfrac{a+b}{2})$ **; ii)** $E(\dfrac{2ab}{a+b})>E(\sqrt{ab})$

SUBIECTUL III

Se considera cubul $ABCDA'B'C'D'$ **si** M,N,P **mijloacele segmentelor** $[AB],[BC]$ **respectiv** $[C'D']$**.**

a) **Demonstrati ca** $(MNP)\parallel(A'BC')$ **; b) Daca** $AB=a$**, calculati** $d(P,MN)$**.**

SUBIECTUL IV

Pe planul triunghiului ABC**, cu** $AB=AC=a, BC=a\sqrt{2}$**, se ridica perpendiculara** DC**, astfel incat** $DC=a$

a) **Calculati** $m[\sphericalangle((DAC),(DBC))]$**; b) Daca** M **si** E **sunt mijloacele segmentelor** $[BC]$ **respectiv** $[DA]$**, demonstrati ca punctul** M **este egal departat de punctele** A,B,C **si** E**; c) Determinati** $tg(\sphericalangle(AC,BD))$**.**

BAREM CLASA A VIII-A

SUBIECTUL I

a) **Inlocuire si calcul direct (2p); b) Daca** (x, y) **este o solutie a ecuatiei, atunci exista si sunt unice** $X, Y \in \mathbb{Z}$ **, astfel incat** $x = X + 1, y = Y + 1$**. Cf. a) avem echivalentele**

$x^2 + y^2 = x + y + xy \Leftrightarrow x^2 + y^2 - x - y - xy = 0 \Leftrightarrow \dfrac{(X-Y)^2 + X^2 + Y^2 - 2}{2} = 0 \Leftrightarrow (X-Y)^2 + X^2 + Y^2$ $= 2$ **(2p);** $((X-Y)^2, X^2, Y^2) \in \{(0,1,1),(1,0,1),(1,1,0)\}$ **(2p);** $(x, y) \in \{(1,2),(2,1),(2,2)\}$ **(1p)**

SUBIECTUL II

a) $y^2 + \dfrac{1}{y^2} - y - \dfrac{1}{y} = \ldots = \dfrac{(y-1)(y^3-1)}{y^2}$ **(1p)**

b) i) $E(\dfrac{a+b}{2}) = \ldots = 2$ **(1p);** $E(\dfrac{2ab}{a+b}) = \dfrac{a}{b} + \dfrac{b}{a}$ **(1p);** $E(\dfrac{2ab}{a+b}) - E(\dfrac{a+b}{2}) = \dfrac{(a-b)^2}{ab} > 0$ **(1p)**

ii) $E(\sqrt{ab}) = \sqrt{\dfrac{a}{b}} + \sqrt{\dfrac{b}{a}}$ **(1p); Daca** $0 < a < b$**, fie** $y = \sqrt{\dfrac{a}{b}}$ **. Atunci** $0 < y < 1$ **si, tinand seama de a), avem:** $E(\dfrac{2ab}{a+b}) - E(\sqrt{ab}) = y^2 + \dfrac{1}{y^2} - y - \dfrac{1}{y} = \dfrac{(y-1)(y^3-1)}{y^2} > 0$ **(2p).**

SUBIECTUL III

a) $MN \parallel AC$ **si** $AC \parallel A'C'$ **(** $ACC'A'$ **paralelogram)** $\Rightarrow MN \parallel A'C'$ **(1p);** $MP \parallel BC'$ **(1p);** $(MNP) \parallel (A'BC')$ **(1p);**

b) **Fie** Q **mijlocul** $[CD]$**;** $PQ \parallel C'C \Rightarrow PQ \perp (ABC)$ **(1p);** $\triangle CQN$ **si** $\triangle BNM$ **sunt dreptunghice si isoscele deci** $m(\sphericalangle MNQ) = 90^\circ$ **(1p);** $d(P, MN) = PN$ **(teorema celor trei perpendiculare) (1p);** $QN = \dfrac{a\sqrt{2}}{2}$ **,** $PQ = a \Rightarrow PN = \dfrac{a\sqrt{6}}{2}$ **(1p).**

SUBIECTUL IV

a) $AC, BC \perp DC \Rightarrow m[\sphericalangle((DAC),(DBC))] = m(\sphericalangle BCA)$ **(1p);**
$\triangle ABC$ **este isoscel si dreptunghic in** A $m(\sphericalangle BCA) = 45^\circ$ **(1p);**

b) $[AM]$ **este mediana in** $\triangle ABC$**, dreptunghic in** $A \Rightarrow MA = MB = MC = \dfrac{a\sqrt{2}}{2}$ **(0,5p);** $AM \perp BC, AM \perp DC \Rightarrow AM \perp (DBC)$ **(1p);** $[ME]$ **este mediana in** $\triangle DMA$**, dreptunghic in** M $DM = \dfrac{AD}{2} = \dfrac{a\sqrt{2}}{2}$ **(0,5p);**

c) **Fie** P **mijlocul lui** $[AB]$**. Se demonstreaza ca:** $PM \parallel AC, PE \parallel BD \Rightarrow m(\angle AC, BD) = m(\angle EPM)$ **(1p);**

$BD = a\sqrt{3}, PE = \frac{a\sqrt{3}}{2}, PM = \frac{a}{2}$ **si, cum** $ME = \frac{a\sqrt{2}}{2} \Rightarrow \triangle MEP$ **este dreptunghic in** M **(1p);** $tg\angle EPM = \sqrt{2}$ **(1p).**