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OLIMPIADA NAȚIONALĂ DE MATEMATICĂ

ETAPA LOCALĂ , CLASA a XI - a

22 FEBRUARIE 2014

SUBIECTUL I

Se considera matricea $A \in M_2(\square),\ A = \begin{pmatrix} 3 & 3 \\ 2 & 2 \end{pmatrix}$.

a) **Sa se calculeze** A^n **,** $n \in \square^*$ **.**

b) **Sa se rezolve ecuatia** $X^{2015} = A,\ X \in M_2(\square)$ **.**

SUBIECTUL II

Sa se calculeze:

a) $\lim\limits_{n\to\infty}(\sqrt[3]{n^3+9n^2+1}+\sqrt[3]{n^3-9n^2+1}-2n)$

b) $\lim\limits_{n\to\infty} n\cdot(\sqrt[3]{n^3+9n^2+1}+\sqrt[3]{n^3-9n^2+1}-2n)$

SUBIECTUL III

a) **Sa se gaseasca doua matrice patratice de ordin doi cu elemente reale cu proprietatea ca** $A^2+B^2=\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$.

b) **Sa se arate ca orice doua matrice patratice de ordin doi cu elemente reale cu proprietatea** $A^2+B^2=\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$ **nu comuta.**

(GM 12/2013)

SUBIECTUL IV

Se considera sirul:

$(x_n)_{n\ge1}$ **de numere reale definit prin** $x_1=\sqrt{\frac{1}{2}}$ **,** $x_{n+1}=\sqrt{\frac{1+x_n}{2}},\ n\ge1$. **Sa se calculeze** $\lim\limits_{n\to\infty} x_1\cdot x_2\cdot\ldots\ldots\cdot x_n$.

(GM 12/2013)

BAREM CLASA XI

SUBIECTUL I

a) $A^2 = 5A$ (1p)

$A^n = 5^{n-1} \cdot A$(0,5p)

Finalizare prin inductie (0,5p)

b) $\det X = 0$ (1p)

$X^2 = t \cdot X, t \in \mathbb{R}, t = Tr(X)$ (1p)

$X^{2015} = t^{2014} \cdot X \Rightarrow X = \frac{1}{t^{2014}} \cdot A$ (1p)

$X^{2015} = \frac{1}{t^{2014 \cdot 2015}} \cdot 5^{2014} \cdot A \Rightarrow t^{2014} = \frac{1}{\sqrt[2015]{5^{2014}}}$ (1p)

$X = \frac{1}{\sqrt[2015]{5^{2014}}} \cdot A$ (1p)

SUBIECTUL II

a) $\lim\limits_{n\to\infty}((\sqrt[3]{n^3+9n^2+1}-n)+(\sqrt[3]{n^3-9n^2+1}-n))$ **(0,5p)**

$\lim\limits_{n\to\infty}(\sqrt[3]{n^3+9n^2+1}-n) = 3$ **(0,5p)**

$\lim\limits_{n\to\infty}(\sqrt[3]{n^3-9n^2+1}-n) = -3$ **(0,5)**

Finalizare(0,5p)

b) $\lim\limits_{n\to\infty} n((\sqrt[3]{n^3+9n^2+1}-(n+3))+(\sqrt[3]{n^3-9n^2+1}-(n-3)))$ **(2p)**

$\lim\limits_{n\to\infty} n(\sqrt[3]{n^3+9n^2+1}-(n+3)) = -9$ (1p)

$\lim\limits_{n\to\infty} n(\sqrt[3]{n^3-9n^2+1}-(n-3)) = -9$ (1p)

Finalizare(1p)

SUBIECTUL III

a) $\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 3 & 2 \end{pmatrix}$(1p)

$\begin{pmatrix} 2 & 3 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & \frac{3}{\sqrt{2}} \\ 0 & 0 \end{pmatrix}^2$ (1p); $\begin{pmatrix} 0 & 0 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \frac{3}{\sqrt{2}} & \sqrt{2} \end{pmatrix}^2$ (1p)

$$A=\begin{pmatrix}\sqrt{2} & \frac{3}{\sqrt{2}}\\ 0 & 0\end{pmatrix}, B=\begin{pmatrix}0 & 0\\ \frac{3}{\sqrt{2}} & \sqrt{2}\end{pmatrix}$$

b) Presupunem, prin reducere la absurd, ca $AB=BA$(1p)

$\det(A^2+B^2)=\det((A+iB)(A-iB)=|\det(A+iB)|^2$ (2p)

$\det\begin{pmatrix}2 & 3\\ 3 & 2\end{pmatrix}<0$ (1p).

SUBIECTUL IV

$x_1=\cos\frac{\pi}{4}$ (1p); $x_2=\cos\frac{\pi}{8}$ (1p); $x_n=\cos\frac{\pi}{2^{n+1}}$ (1p); $x_1\cdot x_2\cdot ......\cdot x_n=\frac{1}{2^n\sin\frac{\pi}{2^{n+1}}}$ (2p);

$\lim\limits_{n\to\infty} x_1\cdot x_2\cdot ......\cdot x_n=\frac{2}{\pi}$ (2p).